MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sofia Tirabassi

Tentamensskrivning i
Topologi, MM8002/SF2721
7.5 hp
2019-05-23

(1) Let $X \subseteq \mathbb{R}^3$ be the subspace $X := \{(x, y, z) \in \mathbb{R}^3 \mid xyz \geq 0\}$
  (a) Determine whether X is open in $\mathbb{R}^3$.
  (b) Determine whether X is closed in $\mathbb{R}^3$.
  (c) Determine the boundary of X, ∂X.
  (d) Determine whether X is connected.
  (e) Determine whether the interior of X, $\mathrm{Int}(X)$, is connected.
  (f) Show that X is contractible. [30 points]

(2) Let $I = [0, 1]$ with the usual (Euclidean) topology. Let also $I_0 = (\frac{1}{3}, \frac{2}{3})$, and denote by $Y := I/I_0$ the space obtained from I collapsing I_0 to a point. Finally denote by $Z := I/\overline{I_0}$ the space obtained from I by collapsing the closure of I_0 to a point.
  (a) Determine whether Y is connected.
  (b) Determine whether Y is compact.
  (c) Determine whether Y is Hausdorff (T_2).
  (d) Show that Z is homeomorphic to I.
  (e) Determine whether Y and Z are homeomorphic. [25 points]

(3) Let G subgroup of $GL_3(\mathbb{R})$ given by matrices of the form

$$\begin{pmatrix} 1 & x & y \\ 0 & 1 & z \\ 0 & 0 & 1 \end{pmatrix}.$$

Endow G with the subset topology induced by the Euclidean topology in $\mathbb{R}^9$.
  (a) Show that G is a topological group which is homemorphic to $\mathbb{R}^3$ (with the Euclidean topology) but not isomorphic $(\mathbb{R}^3, +)$.
  (b) Let Γ be the subgroup of G constituted by matrices with integer coefficient. Then Γ acts on G by left multiplication. Show that the quotient map $G \to G/\Gamma$ is a covering space. (**Hint:** it might be easier if you identify G with $\mathbb{R}^3$). [15 points]

(4) Let X be the connected sum of two tori minus one point, that is

$$X := (T \sharp T) \backslash \{P\}.$$

Compute the fundamental group of X. [10 points]

(5) Formulate and prove the homotopy invariance of the fundamental group. [20 points]