MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Jonathan Rohleder

Tentamensskrivning i
Matematik III Komplex Analys
7.5 hp
30 January 2019

No calculators, books, or other resources allowed. Max score on each problem is 5p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks.

1. Determine the value of the integral
$$\int_0^\infty \frac{1}{(1+x^2)^3}\,\mathrm{d}x.$$
2. (a) Determine the order of the pole of $f(z) = \frac{1}{(\sin z + z)^2}$ at $z = 0$.

   (b) Assume that the analytic function $f(z)$ has a pole of order m at the point z_0. Prove that $f'(z)$ has a pole of order $m+1$ at z_0.
3. Let γ be a directed smooth curve with initial point α and terminal point β. Show that
$$\int_\gamma z\,\mathrm{d}z = \frac{\beta^2 - \alpha^2}{2}.$$
   Which result does this yield if γ is a closed curve? Give an alternative explanation for the result for a closed curve.
4. Calculate all Laurent series expansions of the function $f(z) = \frac{1}{2z^2+4z-6}$ centered at $z_0 = 1$.
5. (a) Use Cauchy's integral formula to determine the value of
$$\oint_{|z|=2} \frac{\cos z}{z^2 - 5z + 4}\,\mathrm{d}z.$$
   (b) Suppose that f is analytic inside and on the unit circle $|z| = 1$ and satisfies $|f(z)| \le M$ for all z with $|z| = 1$. Verify that $|f'(i/2)| \le 4M$ holds.
6. Find a conformal mapping of the first quadrant onto itself which maps the point $1+i$ to the point $2+i$.

Exams will be returned on 1 February 2019 at 3:30 pm in room 322, building 6, and will be stored in the students' office afterwards.