MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Jonathan Rohleder

Tentamensskrivning i
Matematik III Komplex Analys
7.5 hp
15 January 2020

**No calculators, books, or other resources allowed. Max score on each problem is 5p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks.**

1. Calculate all Laurent series expansions of the function

$$f(z) = \frac{1}{z^2 - (4+i)z + 3 + 3i}$$

centered at $z_0 = 0$.

2. Compute the contour integral

$$\int_\Gamma \left( |z - 1 + i|^2 - z \right) \mathrm{d}z,$$

where $\Gamma$ is the positively oriented upper semicircle of radius one centered at $1 - i$.

3. Use residue calculus to determine the value of the integral

$$\int_0^{2\pi} \frac{2}{\cos x + 3} \, \mathrm{d}x.$$

4. (a) Show that $u(x, y) = \operatorname{Re} f(z)$ is harmonic for any analytic function $f$, where $z = x + iy$.

(b) Prove that the harmonic conjugate of a given harmonic function $u$ is unique up to a constant: If $v_1, v_2$ are two harmonic conjugates of $u$ then $v_1 - v_2$ is constant.

5. Sketch the region

$$S = \left\{ z = re^{i\varphi} : r > 0, 0 < \varphi < \frac{3\pi}{4} \right\}$$

in the complex plane and find a conformal mapping of the open disk of radius one centered at the origin onto $S$ that maps the origin onto the point $e^{\frac{3\pi}{8}i}$.

6. Compute the integral

$$\iint_{\partial_0 P} \frac{3}{1 - 2zw} \, \mathrm{d}z \, \mathrm{d}w,$$

where $\partial_0 P = \{(z, w) : |z| = |w| = 1\}$ is the distinguished boundary of the unit polydisk centered at the origin, taken with the usual orientation.

Exams will be returned on 16 January 2020 at 3 pm in room 414, building 6, and will be stored in the students' office afterwards.