MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sven Raum

Tentamensskrivning i
Ordinary Differential Equations
7.5 hp
May 27, 2019

No calculators, books, or other resources allowed. Max score is 30p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks.

1. **(6p)** Find all solutions to the differential equation $xy' + (x-2)y = x^4$.
2. **(4p)** Determine the general, real solution to the system
$$\begin{Bmatrix} x' = & 2x - y \\ y' = & x \end{Bmatrix}$$
3. **(7p)** Use the power series methods to find the solution to the initial value problem
$$\begin{Bmatrix} x^2y'' + xy' + x^2y = 0 \\ y(0) = 1 \\ y'(0) = 0 \end{Bmatrix}$$
4. **(7p)** (a) Determine all critical points of the autonomous system
$$\begin{Bmatrix} x' = -e^x y \\ y' = y^2 - x^2 - 2y + 2x \end{Bmatrix}$$
(b) Investigate whether these critical points are asymptotically stable, stable or unstable.
5. **(6p)** Consider the boundary value problem
$$u'' + 2u' + u = 0 \quad \text{in } [0,1], \qquad u(0) + u'(0) = 0, \quad u(1) - u'(1) = 0. \tag{$*$}$$
(a) Show that this boundary value problem has a unique solution.
(b) Find a Sturm-Liouville boundary value problem with the same solutions as $(*)$.