MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sven Raum

Tentamensskrivning i
Ordinary Differential Equations
7.5 hp
August 21, 2019

No calculators, books, or other resources allowed. Max score is 30p; grade of E guaranteed at 15p. Appropriate amount of details required for full marks.

1. **(10p)** Consider the system of linear differential equations

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}' = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \begin{pmatrix} e^x \\ 0 \end{pmatrix} \qquad (*)$$

   and solve the following questions.

   (a) Find all solutions to the differential equation $y' = y + e^t$.

   (b) Find a fundamental matrix of the homogeneous system associated with $(*)$.

   (c) Calculate the Wronskian associated with the fundamental matrix you found.

   (d) Apply the Duhamel formula to find a particular solution of $(*)$.

2. **(8p)** Use the Laplace transform methods to find the solution to the initial value problem

$$\begin{Bmatrix} y'' + y = x \\ y(0) = 0 \\ y'(0) = 0 \end{Bmatrix}$$

3. **(6p)**

   (a) Consider a general autonomous system of differential equations

$$\begin{Bmatrix} x' = F(x,y) \\ y' = G(x,y) \end{Bmatrix}$$

   and define the following terms:

   - a "critical point" of the system,
   - a "stable" critical point of the system, and
   - an "asymptotically stable" critical point of the system.

   (b) Consider the autonomous system

$$\begin{Bmatrix} x' = \sin(x) + \sin(y) \\ y' = (x+1)(y-1) + 1 \end{Bmatrix}$$

   Find all its critical points and determine for each of them whether it is stable, asymptotically stable or unstable.

4. **(8p)**

   (a) Provide an example of a 2nd order boundary value problem with von Neumann type boundary conditions that does not have a unique solution.

   (b) State the Fredholm alternative for Sturm-Liouville type boundary value problems.

(c) Rewrite the differential expression $L\,u = u'' - 2u' + u$ in the form of a Sturm-Liouvile problem and find a fundamental solution that is not a Green function.

Corrected exams can be consulted on 29th August, 2019 between 9 - 10 am in room 110, building 6A, and will be stored in the students' office afterwards.