MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examiner: Sven Raum
Instructor: Corentin Léna

Exam in
Ordinary differential equations
7.5 hp
26th August 2020
9:00 to 16:00

Please read carefully the general instructions:

- This is an open book exam.
- In all your solutions show your reasoning and calculation, explaining carefully what you are doing. Justify your answers.
- Use natural language, not just mathematical symbols.
- Use clear and legible writing. Write preferably with a ball-pen or a pen (black or dark blue ink).
- Upload the exam on the course page before 16 o'clock
- A maximum score of 100 points can be achieved. A score of at least 50 points will ensure a pass grade if item 0 is completed.

GOOD LUCK!

0. **Mandatory, but gives no points.**
   The PDF document that contains your home exam should start by you writing the following sentence:

   I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person. This means that I have for example not discussed the solutions or the home exam with any other person.

1. **Systems of differential equations** (20 points)
   Find the general solution of the system of differential equations

$$X' = AX$$

   where A is the matrix

$$\begin{pmatrix} -2 & 1 & -1 \\ -3 & 2 & -1 \\ -1 & 1 & 0 \end{pmatrix}.$$

2. **Higher order differential equations** (20 points)
   Solve the differential equation with boundary conditions

$$\begin{aligned} f''(x) + f'(x) - 2f(x) &= 3e^x - 18xe^{-2x} \\ f(0) &= 0 \\ f'(-1) &= -6e^2. \end{aligned}$$

3. **Laplace transform** (20 points)
   Solve the following initial value problem by means of the Laplace transform and express its solution as an elementary function.

$$\begin{aligned} f''(x) + f(x) &= x^2 \\ f(0) &= 0 \\ f'(0) &= 0. \end{aligned}$$

4. **Autonomous systems of differential equations** (20 points)
   For the following autonomous system, find all equilibrium points and determine whether they are asymptotically stable, stable or unstable

$$\begin{Bmatrix} x' = & \frac{1}{2}\sin^2(x) + y \\ y' = & x^2 - y \end{Bmatrix}$$

5. **Boundary value problems** (20 points)
   Show that for any $\lambda > 0$ the following boundary value problem has a unique solution. Express this solution as an elementary function.

$$\begin{aligned} u'' &= \lambda u \text{ on } [0,1] \\ u(0) &= 0 \\ u'(1) &= 1. \end{aligned}$$