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Suggested solutions:
Econometric methods (MT5014)
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9-14

Problem 1

(A) A time series $\{r_t\}$ is weakly stationary if neither $E(r_t)$ nor $Cov(r_t, r_{t-l})$ depends on t (where l is an arbitrary integer); see also Tsay p. 30.

(B) Since the coefficient in front of r_{t-1} is 0.4, which satisfies $|0.4| < 1$, this time-series is weakly stationary (see Tsay p. 39). Hence by taking expectation, i.e $E(r_t) = E(0.1 + 0.4r_{t-1} + a_t)$, and using weak stationary and $E(a_t) = 0$ we find $E(r_t) = 0.1 + 0.4E(r_t)$. Solving for $E(r_t)$ gives $E(r_t) = 0.1/0.6 = 1/6$.

Similarly, we find $V(r_t) = 0.4^2 V(r_{t-1}) + V(a_t)$. Using weak stationary and $V(a_t) = 1$ we find $V(r_t) = 1/(1 - 0.4^2)$.

(C) Consider the AR(1) time-series in **(B)** but change the coefficient 0.4 to 1 and the coefficient 0.1 to 0, then you have a definition of random walk; see also Tsay p. 72.

Problem 2

This relies on Tsay around p. 54-56.

(A) The forecast is $\hat{r}_3(1) = 0.3 + 0.8r_3 = 0.3 + 0.8 \cdot 0.2 = 0.46$.

(B) The forecast is $\hat{r}_3(2) = 0.3 + 0.8\hat{r}_3(1) = 0.3 + 0.8 \cdot 0.46 = 0.668$. The forecast error is,

$$e_3(2) = a_5 + 0.8a_4.$$

Hence, the associated variance is $V(e_3(2)) = V(a_5) + V(0.8a_5) = (1 + 0.8^2)V(a_t) = 0.164$.

(C) The 1 step ahead forecast is,

$$\hat{r}_3(1) = 0.3 + 0.8r_3.$$

The 2 step ahead forecast is

$$\hat{r}_3(2) = 0.3 + 0.8\hat{r}_3(1) = 0.3 + 0.8(0.3 + 0.8r_3).$$

The 3 step ahead forecast is

$$\hat{r}_3(3) = 0.3 + 0.8\hat{r}_3(2) = 0.3 + 0.8(0.3 + 0.8(0.3 + 0.8r_3)).$$

The 4 step ahead forecast is

$$\hat{r}_4(3) = 0.3 + 0.8\hat{r}_3(3) = 0.3 + 0.8(0.3 + 0.8(0.3 + 0.8(0.3 + 0.8r_3))).$$

Continuing in this way clearly gives

$$\lim_{n\to\infty} \hat{r}_3(n) = 0.3(1 + 0.8 + 0.8^2 + ...)$$

Recall that $(1 + 0.8 + 0.8^2 + ...) = 1/(1 - 0.8)$. Recall the general formula for $E(r_t)$ for a weakly convering AR(1), which in this particular case gives $E(r_t) = 0.3/(1 - 0.8)$. It follows that $\lim_{n\to\infty} \hat{r}_3(n) = E(r_t)$ and we are done.

Problem 3

We use the Chow test of structural change (p. 76-77 in the compendium). The F-statistic is

$$F = \frac{(18.3 - (9.1 + 4.3))/3}{(9.1 + 4.3)/(100 - 2 * 3)} = 11.4577.$$

Since the F-statistic is this large we can reject the hypothesis that the parameters are the same in the two sub-periods at a very small significance level (the exact significance level at which the hypothesis can be rejected can be looked up in a table for the F distribution). See p. 76-77 in the compendium for further details.

Problem 4

(A) $\boldsymbol{a}_t$ should be a sequence of serially uncorrelated random vectors with mean zero and a covariance matrix $\boldsymbol{\Sigma}$ (which is typically positive definite) – Tsay p. 399.

(B) It is always weakly stationary (due to the assumption that the covariance matrix $\boldsymbol{\Sigma}$ exists) – Tsay p. 417-418.

(C) See Tsay p. 419.

(D) A feedback relationship – Tsay p. 419.

Problem 5

(A) Heteroskedasticity.

(B) It is unbiased. This can be seen by using the formula for the OLS estimate and the assumption that the expected values of the errors $\tilde{\varepsilon}_j$ are zero. Hint: see the last equation on p. 70 in the compendium.

(C) The GLS formula is (p. 89 in the compendium)

$$\hat{\boldsymbol{\beta}}_{GMM} = (\boldsymbol{X}^T \boldsymbol{\Omega}^{-1} \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{\Omega}^{-1} \boldsymbol{Y}$$

where for the model in the problem we have $\boldsymbol{\Omega} = \text{diag}(e^{2X_1}, ..., e^{2X_n})$, and

$$\boldsymbol{X} = \begin{bmatrix} 1 & X_1 \\ \vdots & \vdots \\ 1 & X_n \end{bmatrix}$$

$$\boldsymbol{Y} = \begin{bmatrix} Y_1 \\ \vdots \\ Y_n \end{bmatrix}$$

(D) It is BLUE (p. 90 in the compendium).

Problem 6

(A) It is not unbiased. To see this use that

$$\hat{\boldsymbol{\beta}}_{OLS} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{Y} = \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T(\mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}) = \boldsymbol{\beta} + \left(\mathbf{X}^T\mathbf{X}\right)^{-1}\mathbf{X}^T\boldsymbol{\varepsilon}$$

and that we have not assumed that $E[\boldsymbol{\varepsilon}|\mathbf{X}]$ is zero.

(B) See p. 116 in the compendium.