STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

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# Solution to exam in Econometric Methods (MT5014), 13 January 2020

## Problem 1

**a)** See in compendium, chapter 2.2.2.
**b)** See definition 3.3, on page 36 in the compendium.
**c)** See page 36 in the compendium.
**d)** It is biased.
**e)** A valid instrument should be not correlated with noise in regression equation and correlated with explanatory variable.

## Problem 2

**a)** Autocorrelation. Estimator of $\boldsymbol{\beta}$ is unbiased and consistent but not effective.
**b)** No. Since $\varepsilon_1,\ldots,\varepsilon_n$ has constant variance, they are homoscedastic.
**c)** AR(1)-process.
**d)** GLS-estimator of $\boldsymbol{\beta}$ in this model is

$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^T\boldsymbol{\Omega}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{\Omega}^{-1}\boldsymbol{Y}$, where $\boldsymbol{\Omega} = \begin{bmatrix} 1 & \theta & \cdots \theta^{n-1} \\ \theta & 1 & \cdots \theta^{n-2} \\ \vdots & \vdots & \ddots \quad \vdots \\ \theta^{n-1} & \theta^{n-2} \cdots & 1 \end{bmatrix}$

and it is BLUE (Theorem 4.1 in compendium). We need to estimate *k*+2 parameters in this model: $\beta_1, \ldots, \beta_k, \sigma, \theta$.
**e)** Since $\varepsilon_t$ is a AR(1) process so $\varepsilon_t = \theta\varepsilon_{t-1} + \tilde{\varepsilon}_t$. We can approximate $\varepsilon_t$ with the residuals $e_t$ and estimate $\theta$ by OLS.

## Problem 3

The unrestricted RSS is $\mathbf{e}^T\boldsymbol{e} = (102-2)26 + (102-2)42 + (52-2)0.30 = 83$ and the restricted RSS is $\mathbf{e}_*^T\mathbf{e}_* = (256-2)0.38 = 96.52$.

The Chow test of structural change is

$$F = \frac{(\mathbf{e}_*^T\mathbf{e}_* - \mathbf{e}^T\mathbf{e})/(2*2)}{\mathbf{e}^T\mathbf{e}/(256-3*2)} = \frac{(96.52-83)/4}{83/250} = 10.181.$$

This is significant at any conventional significance levels and we reject the hypothesis that all income groups have the same expenditure function.

## Problem 4

See in the textbook of Tsay chapter 2.6.1.

## Problem 5

**a)** See page 59 in the textbook of Tsay.
**b)** See pages 62-63 in the textbook of Tsay.
Multistep-ahead forecasts go to the mean after the first two steps, the variance of the forecast errors go to the variance of $Y_t$ after two steps.

## Problem 6

**a)** See page 391 in the textbook of Tsay.
**b)** See the end of page 391 in the textbook of Tsay.
**c)** Formula (8.8) on page 399 in the textbook of Tsay.
**d)** The $k$ eigenvalues of $\mathbf{\Phi}$ must be less than 1 in modulus (see page 402 in the textbook of Tsay).
**e)** This is a structural form.