STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

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Solution to exam in Econometric Methods (MT5014), 13 February 2020

Problem 1

a) See in compendium, chapter 2.2.2.
b) Unbiasedness does not depend on the size of the data sample, see page 38 in the compendium.
c) Residuals are $\boldsymbol{e} = \boldsymbol{Y} - \widehat{\boldsymbol{Y}}$ and see point 1. on page 28 in the compendium.
d) 1. Heteroscedasticity. $\widehat{\boldsymbol{\beta}}$ are unbiased, consistent but not effective.
2. Autocorrelation. $\widehat{\boldsymbol{\beta}}$ are unbiased, consistent but not effective.
3. Collinearity. The matrix $\boldsymbol{X^T X}$ is not invertible so we cannot estimate $\boldsymbol{\beta}$.
4. No stable $\widehat{\boldsymbol{\beta}}$ over all observations (no parameter constancy).

Problem 2

a) Heteroscedasticity. Estimator of $\boldsymbol{\beta}$ is unbiased and consistent but not effective.
b) It is unbiased. See the last equation on page 70 in the compendium.
c) GLS-estimator of $\boldsymbol{\beta}$ in this model is
$\widehat{\boldsymbol{\beta}} = (\boldsymbol{X}^T\boldsymbol{\Omega}^{-1}\boldsymbol{X})^{-1}\boldsymbol{X}^T\boldsymbol{\Omega}^{-1}\boldsymbol{Y}$, where $\boldsymbol{\Omega} = \text{diag}(X_1^2, \dots, X_n^2)$.
d) It is BLUE (Theorem 5.1 in compendium).

Problem 3

The unrestricted RSS are $\mathbf{e}_1^T\mathbf{e}_1 = 30 - [10\ 20]\begin{bmatrix} 20 & 20 \\ 20 & 25 \end{bmatrix}^{-1}\begin{bmatrix} 10 \\ 20 \end{bmatrix} = 5$
and $\mathbf{e}_2^T\mathbf{e}_2 = 24 - [8\ 20]\begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix}^{-1}\begin{bmatrix} 8 \\ 20 \end{bmatrix} = 3.2.$

To get the restricted RSS we need the design matrix for whole sample

$\mathbf{X}_*^T\mathbf{X}_* = \begin{bmatrix} 20 & 20 \\ 20 & 25 \end{bmatrix} + \begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix} = \begin{bmatrix} 30 & 30 \\ 30 & 45 \end{bmatrix}$, $\mathbf{X}_*^T\mathbf{Y}_* = \begin{bmatrix} 10 \\ 20 \end{bmatrix} + \begin{bmatrix} 8 \\ 20 \end{bmatrix} = \begin{bmatrix} 18 \\ 40 \end{bmatrix}$,
$\mathbf{Y}_*^T\mathbf{Y}_* = 30 + 24 = 54$

The Chow test of structural change is

$$F = \frac{(\mathbf{e}_*^T\mathbf{e}_* - (\mathbf{e}_1^T\mathbf{e}_1 + \mathbf{e}_2^T\mathbf{e}_2))/\text{k}}{(\mathbf{e}_1^T\mathbf{e}_1 + \mathbf{e}_2^T\mathbf{e}_2)/(\text{n} - 2\text{k})} = \frac{(10.933 - 5 - 3.2)/2}{(5 + 3.2)/26} = 4.333.$$

The 5% critical value is $F_{0.95}(2, 26) = 3.37$. We reject the hypothesis (at 5% significance level) that urban and rural areas have the same structure.

Problem 4

Let $Y_t = \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \varepsilon_t - \theta_1 \varepsilon_{t-1}$ $(*)$.

By squaring both sides of the above equation and taking expectations we get $\gamma_0(1-\phi_1^2-\phi_2^2) = 2\phi_1\phi_2\gamma_1 + \sigma_\varepsilon^2(1-2\phi_1\theta_1+\theta_1^2)$ where $\gamma_0 = E[Y_t^2] = V(Y_t)$.
To get the above you should remember or show that $E[Y_{t-1}\varepsilon_{t-1}] = \sigma_\varepsilon^2$. Then multiplying both sides of (*) by successive lags of itself and taking expectations we get $\gamma_1(1-\phi_2) = \phi_1\gamma_0 + \theta_1\sigma_\varepsilon^2$ and $\gamma_k = \phi_1\gamma_{k-1} + \phi_2\gamma_{k-2}$ for $k = 2, 3, \ldots$ where $\gamma_l = Cov(Y_t, Y_{t-l}), l > 0$.
And the autocorrelations are $\rho_0 = 1, \rho_1 = \frac{\gamma_1}{\gamma_0}, \rho_k = \frac{\gamma_k}{\gamma_0} = \phi_1\rho_{k-1} + \phi_2\rho_{k-2}, k = 2,3, \ldots$

Problem 5

a) See page 59 in the textbook of Tsay.
b) See page 62 in the textbook of Tsay.

Problem 6

a) See page 401 in the textbook of Tsay.
b) See page 402 in the textbook of Tsay.
c) See page 403 in the textbook of Tsay.