STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

Exam
MT5014
13 January 2020

Exam: Econometric methods (MT5014)

13 January 2020, 9:00-14:00

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Allowed aid: Calculator (provided by the Department) and table with F-quantiles which is attached to the writing.
Return of exam: To be announced via the course web page or the course forum.

The exam consists of six problems. Each correctly solved problem gives 10 points.
Important:

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- **Write your code number (but no name) on each sheet.**

Preliminary grading:

	A	B	C	D	E
Points	54	48	42	36	30

Good luck!

Problem 1

Consider the classical multiple linear regression model $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$.

a) Give the assumptions in this model. (2p)
b) What does it mean that OLS estimator of $\boldsymbol{\beta}$ is BLUE? (2p)
c) Calculate the variance-covariance matrix of $\hat{\boldsymbol{\beta}}$. (3p)
d) If the assumption $E[\boldsymbol{\varepsilon}|\boldsymbol{X}] = \boldsymbol{0}$ is not fulfilled what are the consequences for the OLS estimator of $\boldsymbol{\beta}$? (1p)
e) If we need to introduce the instrumental variable state the two qualities that a valid instrument must have. (2p)

Problem 2

Consider the regression model $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$, where $\boldsymbol{Y}$ is n x 1-vector, $\boldsymbol{X}$ is n x k-matrix ($k < n$), $\boldsymbol{\beta}$ is unknown k x 1-vector and $\boldsymbol{\varepsilon} = (\varepsilon_1,\ldots,\varepsilon_n)^T$ where $\varepsilon_1,\ldots,\varepsilon_n$ are Normal distributed random variables with the following variance-covariance matrix:

$$\Sigma = \sigma^2 \begin{bmatrix} 1 & \rho_1 & \cdots & \rho_{n-1} \\ \rho_1 & 1 & \cdots & \rho_{n-2} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{n-1} & \rho_{n-2} & \cdots & 1 \end{bmatrix}$$

a) Which specification error is presented in the model? What are the consequences for the OLS estimator of $\boldsymbol{\beta}$? (2p)

b) Are ε_t heteroscedastic? (1p)

c) If $\rho_d = \theta^d$ for $|\theta| < 1$ and $d = 1,\ldots,n-1$, so it is said that ε_t constitutes a particular type of stationary process. Which one? (2p)

d) Assume that $\rho_d = \theta^d$ for $|\theta| < 1$ and $d = 1,\ldots,n-1$. Give GLS-estimates of $\boldsymbol{\beta}$-parameters in this model. Are they still BLUE? How many parameters do you need to estimate in this model? (3p)

e) As θ is usually unknown it must usually be estimated. Describe a method with which you can estimate θ. (2p)

Problem 3

A study of vacation expenditures in relation to income was based on data for 256 households, which were grouped into three separate income classes. Log linear regressions (with an intercept term) were computed for each income group and for all households with the following results:

Household Income	Regression slope	Variance estimate of noise in regression	Number of households
Low income	0.02	0.26	102
Middle income	0.09	0.42	102
High income	0.14	0.30	52
All households	0.07	0.38	256

Test whether the expenditure function is the same for all income groups. (10p)

Problem 4

Assuming weak stationarity calculate the autocorrelation function (ACF) for Autoregressive Moving Average ARMA(1,1) model. You are allowed to assume that the constant in the model is equal to zero. (10p)

Problem 5

Consider the Moving-Average model MA(2): $Y_t = \theta_0 + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2}$
with the usual assumptions i.e. $\theta_0, \theta_1, \theta_2$ are constants and $\{\varepsilon_t\}$is a white noise series.

a) Calculate the expected value (mean) and the variance of the model. (3p)
b) Suppose that we are in time index h and are interested in forecasting $Y_{h+l}, l \geq 1$.
Let $\hat{Y}_h(l)$ be the forecast of Y_{h+l}, and $e_h(l) = Y_{h+l} - \hat{Y}_h(l)$ the forecast error.
Calculate the limit of the multistep-ahead forecast of the MA(2) model and the variance of the forecast error. (7p)

Problem 6

a) Define the lag-*l* cross-covariance matrix $\mathbf{\Gamma}_l$ of time series $\mathbf{Y}_t = (Y_{1t}, \dots, Y_{kt})^T$.
Is this cross-covariance matrix symmetric? Explain why it is or it is not symmetric. (3p)
b) Explain why it suffices in practice to consider the cross-correlation matrices $\boldsymbol{\rho}_l$ for $l \geq 0$ instead for all *l*. (2p)

c) Give the definition of the Vector Autoregressive VAR(1) model in reduced form. (2p)

d) When the VAR(1) time series is weakly stationary? (2p)
No proof has to be provided for your answer.

e) Consider the following Vector Autoregressive model VAR(1):

$$\begin{bmatrix} 1 & b_{12} \\ b_{21} & 1 \end{bmatrix} \begin{bmatrix} Y_{1t} \\ Y_{2t} \end{bmatrix} = \begin{bmatrix} \gamma_{11} & \gamma_{12} \\ 0 & \gamma_{22} \end{bmatrix} \begin{bmatrix} Y_{1,t-1} \\ Y_{2,t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix}$$

in which $V(\varepsilon_{1t}) = \sigma_1^2, V(\varepsilon_{2t}) = \sigma_2^2$, $Cov(\varepsilon_{1t}, \varepsilon_{2t}) = 0$ and $b_{12}, b_{21}, \gamma_{11}, \gamma_{12}$ are constants.
Is this VAR(1) model written in structural or reduced form? (1p)