STOCKHOLMS UNIVERSITET
MATEMATISKA INSTITUTIONEN
Avd. Matematisk statistik

Exam
MT5014
13 February 2020

Exam: Econometric methods (MT5014)

13 February 2020, 9:00-14:00

Examiner: Joanna Tyrcha, phone: 16 45 67, e-mail: joanna@math.su.se
Allowed aid: Calculator (provided by the Department) and table with F-quantiles which is attached to the writing.
Return of exam: To be announced via the course web page or the course forum.

The exam consists of six problems. Each correctly solved problem gives 10 points.
Important:

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- **Write your code number (but no name) on each sheet.**

Preliminary grading:

	A	B	C	D	E
Points	54	48	42	36	30

Good luck!

Problem 1

Consider the classical multiple linear regression model $\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$. Let $\widehat{\boldsymbol{Y}} = \boldsymbol{X}\widehat{\boldsymbol{\beta}}$ where $\widehat{\boldsymbol{\beta}}$ is estimator obtained by the method of least squares.

a) Give the assumptions in this model. (2p)
b) If you calculate $\widehat{\boldsymbol{\beta}}$ using a sample of 30 observations, then will this estimator be unbiased? Explain. (2p)
c) What are the residuals in the model? Do they add up to zero? Show it. (2p)
d) State possible specification errors and the consequences they will have for $\widehat{\boldsymbol{\beta}}$. (4p)

Problem 2

Consider the regression model $Y_j = \beta_1 + \beta_2 X_j + \varepsilon_j,\ j = 1, \dots n$, with $V(\varepsilon_j) = \sigma_j^2 = \sigma^2 X_j^2$.

a) Which specification error is presented in the model? What are the consequences for the OLS estimator of $\boldsymbol{\beta}$? (2p)

b) Show whether the OLS estimator is unbiased or not. (3p)

c) Give a formula for GLS estimator for this particular model. (3p)

d) In what sense is the GLS estimator better than the OLS estimator for this model. (2p)
No argument has to be provided for your answer.

Problem 3

The usual two-variable (one variable plus intercept) linear regression model is postulated, and a sample of 20 observations is drawn from an urban area and another sample of 10 observations from a rural area. The sample information in raw form is summarized as follows:
Urban

$$\boldsymbol{X}^T\boldsymbol{X} = \begin{bmatrix} 20 & 20 \\ 20 & 25 \end{bmatrix} \quad \boldsymbol{X}^T\boldsymbol{Y} = \begin{bmatrix} 10 \\ 20 \end{bmatrix} \quad \boldsymbol{Y}^T\boldsymbol{Y} = 30$$

Rural

$$\boldsymbol{X}^T\boldsymbol{X} = \begin{bmatrix} 10 & 10 \\ 10 & 20 \end{bmatrix} \quad \boldsymbol{X}^T\boldsymbol{Y} = \begin{bmatrix} 8 \\ 20 \end{bmatrix} \quad \boldsymbol{Y}^T\boldsymbol{Y} = 24$$

Test the hypothesis that the same relationship holds in both urban and rural areas. (10p)

Problem 4

Assuming weak stationarity calculate the autocorrelation function (ACF) for Autoregressive Moving Average ARMA(2,1) model. You are allowed to assume that the constant in the model is equal to zero. (10p)

Problem 5

Consider the Moving-Average model MA(1): $Y_t = \theta_0 + \varepsilon_t - \theta_1 \varepsilon_{t-1}$
with the usual assumptions i.e. θ_0, θ_1 are constants and $\{\varepsilon_t\}$is a white noise series.

a) Calculate the expected value (mean) and the variance of the model. (3p)
b) Suppose that we are in time index h and are interested in forecasting $Y_{h+l}, l \geq 1$.
   Let $\hat{Y}_h(l)$ be the forecast of Y_{h+l}, and $e_h(l) = Y_{h+l} - \hat{Y}_h(l)$ the forecast error.
   Calculate the limit of the multistep-ahead forecast of the MA(1) model and the variance of the forecast error. (7p)

Problem 6

Consider the Vector Autoregressive VAR(1) model $\boldsymbol{Y}_t = \boldsymbol{\phi}_0 + \boldsymbol{\Phi}\boldsymbol{Y}_{t-1} + \boldsymbol{\varepsilon}_t$ with the usual assumptions i.e. $\boldsymbol{\phi}_0$ is a k-dimensional vector, $\boldsymbol{\Phi}$ is a k x k matrix and $\{\boldsymbol{\varepsilon}_t\}$ is a sequence of serially uncorrelated random vectors with mean zero and covariance matrix $\boldsymbol{\Sigma}$.

a) Calculate expected value of $\mathbf{Y}_t$. (2p)
b) Calculate covariance-variance matrix of $\mathbf{Y}_t$. (4p)
c) Show that $\boldsymbol{\Gamma}_l = \boldsymbol{\Phi}\boldsymbol{\Gamma}_{l-1}$ where $\boldsymbol{\Gamma}_j$ is the lag-j cross-covariance matrix of $\mathbf{Y}_t$ and $l > 0$. (4p)