STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam:
Econometric methods (MT5014)
2021-01-07

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Return of exam: See the course web page.

Information due to this being a home exam:

Complete instructions are found on `https://kurser.math.su.se/course/view.php?id=932` under the headline "HOME EXAM!!!". The following is a summary of the most important instructions:

- This home exam should be handed in on the course webpage (`https://kurser.math.su.se/course/view.php?id=932`) **today (i.e. at the day of the exam) at the latest at 17:00 (deadline)**.
- The solutions to this home exam should be handed in in PDF format (i.e. one PDF file). There are no restrictions regarding what your PDF should contain. For example, the PDF may be based on a Word document, a Latex document, or scanned nicely handwritten solutions. If you plan on "scanning" handwritten solutions using your mobile phone, I suggest downloading and using a "scanning app". If you scan and thereby obtain several PDF files, then there are many programs that can be used to merge several PDF files into one PDF file.
- Write your anonymization-code (anonymiseringskoden) on each page of your solutions. Name your PDF file using your anonymization-code.
- When writing the home exam you may use any literature and computer program.
- Your solutions should be of the same type as for usual exams (i.e. not of "thesis type") .

The exam consists of six problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.

Preliminary grading:

A	B	C	D	E
54	48	42	36	30

Good luck!

Problem 0

The PDF document that contains your home exam should start by you writing the following sentence:

I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person. This means that I have for example not discussed the solutions or the home exam with any other person.

Problem 1

Consider the regression model

$$Y_j = \beta_0 + \beta_1 X_{1j} + \beta_2 X_{2j} + \beta_3 X_{3j} + \beta_4 X_{4j} + \beta_5 X_{5j} + \varepsilon_j$$

which is assumed to satisfy the classical assumptions. You have a sample of size 100 and you want test if

$$\beta_3 - 3\beta_2 = 3.$$

You decide to do this using an F-test for which the test-statistic is calculated using the RSS of two different regression models.

Which two regression models should you fit to find the relevant RSS values? How should you use these RSS values to test the hypothesis?

(10 p)

Problem 2

Consider the time series

$$x_t = a + bx_{t-1} + c\epsilon_t + d\epsilon_{t-1}$$

where $\{\epsilon_t\}$ is a Gaussian white noise series with variance 1 and mean 0.

(A) Assuming $\{x_t\}$ is weakly stationary, find $E[x_t]$ and $V[x_t]$. (4 p)

(B) If $b = d = 0$, is $\{x_t\}$ a strictly stationary series? (Do not forget to motivate your answer). (3 p)

(C) Find the distribution of $x_{t+2}|x_t, \epsilon_t$ (in other words, the distribution of x_{t+2} conditioned on x_t and ϵ_t). (3 p)

Problem 3

Consider the regression model

$$Y_j = \alpha + \beta X_j + \varepsilon_j \tag{1}$$

which is assumed to satisfy the classical assumptions. Unfortunately, there is a measurement error, in the way that instead of observing X_j you observe

$\tilde{X}_j = aX_j$, where $a > 0$ is a constant (note that if $a = 1$ then there is no measurement error).

Find the OLS estimates $\hat{\alpha}$ and $\hat{\beta}$ given this measurement error. Clarify the relationship between each of these estimates and the corresponding usual (without measurement error) OLS estimates for (1).

Hint: Start by writing the fitted regression with measurement error as $Y_j = \hat{\alpha} + \hat{\beta}\tilde{X}_j + e_j$. (10 p)

Problem 4

Consider the ARMA time-series model

$$r_t = \phi_0 + 0.4r_{t-1} + \theta a_{t-1} + a_t$$

where a_t is white noise, $E(a_t) = 0$ and $V(a_t) = \sigma^2$. Suppose $r_8 = 0.4$ and $a_8 = 0.1$.

(A) Set $\phi_0 = 0.2$ and $\theta = 1$. Find the 1 step ahead forecast. (2 p)

(B) Set $\phi_0 = 0.2$ and $\theta = 1$. Find the 2 step ahead forecast and the variance of the associated forecast error. (4 p)

(C) Set $\phi_0 = \theta = 0$. Find the value of $\lim_{l\to\infty} V(e_8(l))$. *Hint: recall that* $e_8(l)$ *is a forecast error.* (4 p)

Problem 5

Consider

$$Y_j = \alpha + \beta X_j + \varepsilon_j, \quad \varepsilon_j = j\tilde{\varepsilon}_j, \quad j = 1, ..., n$$

where $\tilde{\varepsilon}_j$ satisfies the classical assumptions, and $V(\tilde{\varepsilon}_j) = 1$.

For this particular model, give a formula (or formulas) for the GLS estimators of the parameters (make sure that it is from your formula clear how to find each of the estimators $\hat{\alpha}_{GLS}$ and $\hat{\beta}_{GLS}$).

Show whether the GLS estimators are unbiased or not.

Find $V(\hat{\beta}_{GLS}|\boldsymbol{X})$ given a sample of size $n = 3$ in case $X_1 = 1, X_2 = 2, X_3 = 4$. (10 p)

Problem 6

Consider a random variable V_j that is uniformly distribution on the interval $[\theta_0, 15]$. Suppose that the following is a random sample of size $n = 10$ from V_j

$$(8.87, 5.99, 5.86, 7.43, 14.03, 11.90, 14.71, 8.01, 4.27, 6.50)$$

Using MM, estimate the parameter θ_0 using the first moment as a moment condition.

Using GMM, estimate the parameter θ_0 using the first and second moment as moment conditions. Use the identity matrix as weighting matrix. *Hint:*

Finding this estimate involves minimizing a certain function. Write this function as $Q_n(\theta), n = 10$. Your final answer should include a statement of this minimization problem (clarified as far as possible) and it should be clear exactly how the GMM estimate relates to this minimization problem; but you do not have to solve this minimization problem.

(10 p)