MMS024 Logic - Exam 2020-08-21 - Solutions

1 (a) Derivable,
$$P_1 \rightarrow P_2$$
 $P_2 \rightarrow P_1$ $P_2 \rightarrow P_1$

$$\frac{1}{P_2 \rightarrow P_1} \rightarrow I_1$$

(b) Denvolle:

 $2 (a) FU(3x_1 + (n_1) = x_2) = \{x_1\}$

(6)
$$FU(U_{n_3} P_{10}(n_7)) \rightarrow (\exists_n x_7 P_{10}(x_3)) = \{n_3, n_7\}$$

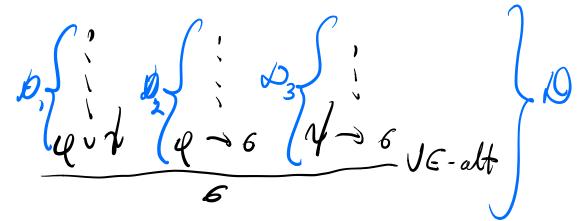
(a) "T" and "TI" me distinct
but logically equivalent farmoles. (b) Suppose 424.
Then in any model V, if q holds, so does 4, i.e. $\varphi \models \psi$. So by completeners, 4 - 1. 5. $\frac{\neg(\forall n, u)}{\neg(\forall n, u)} \rightarrow I_1$ $(\exists n, \neg y) \rightarrow \neg(\forall n, y)$

6. (a) $N, v = P(n, n_1) \rightarrow P(a_1, x_1)$ € f 0<0, then 0<0 which is true, so $[P_{i}(x_{i},x_{i})\rightarrow P_{i}(x_{i},x_{i})]^{N_{i}}=1$ (b) $N_{i} = \forall n_{i} (P_{i}(n_{i}, n_{i}) \rightarrow P_{i}(n_{i}, n_{i})$ € for all NEN, if N=0 then 0<1 which is again true (since 'n20' is always felse) so $[\neg (P,(\alpha_1,\alpha_2)) \rightarrow P,(\alpha_2,\alpha_1)]$ =1. (In, P,(ni)) - (In, P,(ni)) is not a tantology, since it fails in many model, e.g. A = < N; seven numbers; > is a counterwold, since there exit even natural numbers,

but not all natural numbers are even.

8. In soundeness, we prove by induction on derivations (4) "for every derivation 10, in any interp. V, it all assumptions of 10 hold, I sound" then the conclusion of 10 holds."

Cose where Do conclude with vE-alt:



IH: D, D, D, all sound.

Let V be some interp, in which all assim of 20 hold. Then all only of D, D, D, D3 hold in V (since none are discharged by ve-att). So by IV, their conduious ext, exo, and $t \rightarrow 6$ all hold in V.

"V=q of" means that eiter VFy or VFA. (5)

If V=q, then since V=q -6, we get

that V=6.

Similarly, if V=4, then sine V=4-10, V=0.

Bither way, V=6, as required.

9. Call these tructues N+, Nx.

(a) "An, n, n, n, (f, (n, n) = f, (n, n) -> 2/2 = x]"

holds in N_f (since addition of any fixed natural is injective)

but not N_x (since multiplication by O is not injective).

(b) " $\exists n, f_i(n_i, x_i) = n,$ "

holds in \mathcal{N}_+ and \mathcal{N}_K (since O+O=O, O.O=O)

but is not a tautology, since it fails

in e.g. the structure $\{N \setminus \{O\}; ; + \}$.

10. For all there, wite that given a structure 6 $A = \langle A_i ; f \rangle,$ is unjective A= ying (=>) + is surjective A = gruy => f is an involution Cie fof=id,). A = Yinnal () (a) Any involution is a bijection (cince it is a two-sided inverse for itself); is also a model so any model of find of ging i i.e. flud = fing. So by completeness, quand + quij.

(b) Tale some injective but non-sujective (7) function, e.g. successor, S.N-M. Then (N; S) shows Vinj & Young, so by soundners, Ginj H Ysunj. (c) Take some bijection that is not an involution, e.g. $f: \mathbb{Z} \rightarrow \mathbb{Z}$ f(u) := n+1Then (Z; ; f) sleaves that You, Young H Ginrol so again by soundness, Ging, faing of find.

11. (a) An inconsistent theory proves everything? (3) if THI, then for every 4, L Y shows that THY,
So T is complete, since for any

q, THY (and also TH74)! (6) The empty theory \$\phi\$ is not complete: neither P, nor TP, is derivable from Ø. (We see Mis by sound wess: the interp. P:=0 shows that \$ # P1, while Pi=1 shows Ø # 7Pi.)

(c) Suppose T max. cons. Then for (9) any q, we will show etter VI q or If TU (4) is consistent, then by maximality, TU (4) = T, i.e. yET, and so Tty. Otherice, if TU(4) is inconsistent, let 0 be some dans of I from Foly? Then of 141 shows that They.