STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

## Exam: Introduction to Finance Mathematics (MT5009), 2021-05-25

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*Return of exam:* To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.

Preliminary grading:

| A | B | C | D | E |
|---|---|---|---|---|
| 46 | 41 | 36 | 30 | 25 |

## Additional information due to this being a home exam:

- See the course webpage (`https://kurser.math.su.se/course/view.php?id=1009`) for complete instructions.
- If anything is unclear or you are experiencing problems during the exam let me know as soon as possible by sending an email or calling me at 070 444 10 07. **If I need to get in touch with you during the exam, I will send a message via one of the forums on the course webpage, so make sure you are checking them during the exam.**

## Good luck!

## Problem 0

**The PDF document that contains your home exam should start by you writing the following sentence:**

*I, the author of this document, hereby guarantee that I have produced these solutions to this home exam without the assistance of any other person. This means that I have for example not discussed the solutions or the home exam with any other person.*

## Problem 1

**(a)** Suppose the term-structure is deterministic and $y(0,1) = 0.09$ and $y(0,2) = 0.1$ (the time step is $h = 1$).

Determine the bond prices $B(0,1)$, $B(0,2)$ and $B(1,2)$.

**(b)** Suppose that the term-structure is random (but that $y(0,1)$ and $y(0,2)$ are still as above).

Now (at time 0) you receive the information that you will receive 100 SEK at time 1. You want to use this money at time 2 and lock in the interest rate between time 1 and time 2 now. When you receive the 100 SEK you invest this amount in the locked in interest rate.

How much will this investment give you at time 2?

(10 p)

## Problem 2

**(a)** Suppose that you want to invest money today in order to get an infinite stream of payments every quarter, starting one quarter from now, of the amount \$100 the first quarter, and then growing at a rate of 2% compounded quarterly (i.e. the second quarter the payment will be $100(1+0.02/4)$, the third quarter $100(1+0.02/4)^2$, etc). Assume that you can invest at a rate of 5% compounded quarterly. How much do you need to invest today?

**(b)** You decide that you can only afford to invest \$4000. If you still want quarterly payments of the same amount as in (a) (growing at the same rate), how many years will you receive payments for?

(10 p)

## Problem 3

Consider a market consisting of three stocks and a risk-free asset. The three stocks have expected returns 0.10, 0.20, and 0.30 and standard deviations 0.20, 0.30, and 0.25 respectively. The correlation between the first and second stock is 0.5, the correlation between the first and third stock is 0.7, and the correlation between the second and third stock is 0.3. The risk-free return in 0.02. Short selling is allowed.

**(a)** Compute the weights of the market portfolio.

**(b)** You want to invest \$1000. If the highest risk you are willing to take (measured in terms of the standard deviation of your portfolio) is \$100, how should you invest your money to obtain the highest expected return?

(10 p)

# Problem 4

Suppose now is time 0 and that you own 10 shares of Investment Inc. (which pay no dividends). Available on the market is also a futures contract (with the underlying being Investment Inc.) with expiry at $T$ and one marking-to-market date $t$, with $0 < t < T$. Assume that the interest rate is constant.

You are not interested in having the risk of the shares but for some reason you cannot sell them; instead you can however invest in the futures contracts.

Derive a formula for the optimal number of futures contracts should you enter into in order to minimize the risk (measured in variance) of your total wealth (i.e. the shares and any wealth due to futures contract) at time $t$. (Make sure that it is clear if you enter into a long or a short position.)

Also, determine the corresponding the minimal risk. (10 p)

# Problem 5

Consider the Black-Scholes model and a European derivative with maturity at $T$ and payoff function

$$G(x) = \ln(x)$$

where $\ln(x)$ is the natural logarithm. Derive an explicit pricing formula for this derivative at time 0.

Repeat the above but for the payoff function

$$\tilde{G}(x) = x^n$$

where $n \in \{1, 2, 3, ...\}$. (10 p)