STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2021-08-19

Examiner: Kristoffer Lindensjö
 E-mail: kristoffer.lindensjo@math.su.se
 Phone: 070 444 10 07

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.

- Answers should be motivated (unless otherwise stated).

- Any assumptions should be clearly stated and motivated.

- Start every problem on a new sheet of paper.

- Clearly number each sheet with problem number and sheet order.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Additional information due to this being a home exam:

- See the course webpage (`https://kurser.math.su.se/course/view.php?id=1009`) for complete instructions.

- If anything is unclear or you are experiencing problems during the exam let me know as soon as possible by sending an email or calling me at 070 444 10 07. **If I need to get in touch with you during the exam, I will send a message via one of the forums on the course webpage, so make sure you are checking them during the exam.**

Good luck!

Problem 0

Problem 1

John, who has just turned 30 years old, wants to set up a do-it-yourself pension fund by making deposits to a bank account attracting interest at 3% compounded monthly. The fund should enable John to withdraw 10 000 kr per month for 10 years, from the month he turns 65 years old.

(A) What is the present value of the future withdrawals from the bank account?

(B) To ensure that he has enough money in the bank account to cover these future withdrawals, John wants to deposit a fixed amount each month, starting now, until one month before his 65th birthday. What amount should he deposit each month?

(10 p)

Problem 2

Consider a two-period binomial model. Let the current price of the underlying be $S(0) = 1$. Suppose $D < R < U$ and that $U = D + 1$ and $R = 0$.

(A) Derive a formula for the price of a European call option with strike price $X = 1$ and maturity at time 2 as a function of D.

(B) Find the price of this option when $D = -0.5$.

(10 p)

Problem 3

Consider a market consisting of three stocks, with expected returns 0.15, 0.13, and 0.25 and standard deviations 0.20, 0.18, and 0.30 respectively. The correlation between the first and second stock is 0.3, the correlation between the first and third stock is -0.2, and the correlation between the second and third stock is 0.7. Short selling is allowed.

(A) What are the weights of the portfolio (consisting of the three stocks) with the lowest variance?

(B) Compute the weights of a portfolio on the minimum variance line, as a function of the expected return μ_V of the portfolio. What are the weights of the portfolio on the minimum variance line that has the expected return 10%?

(10 p)

Problem 4

Consider a Black-Scholes financial market and European derivative with one year left until maturity and payoff function

$$I_{\{x \geq 5\}}.$$

(A) Describe/interpret the derivative in financial terms with one sentence.

(B) Derive a pricing formula (as explicit as possible) for the derivative.

Hint: recall that the indicator function $I_{\{x \geq 5\}}$ takes the value 1 in case $x \geq 5$ and the value 0 otherwise.

(10 p)

Problem 5

Consider a Black-Scholes financial market and European call option. Let $r = 0.1$, $\mu = 0.15$, $\sigma = 0.3$, $S(0) = 100$, $X = 50$ and $T = 1$. The current time is 0.

(A) Find the price of the call option, $C_E(0)$.

(B) Find the VaR (with 95 % confidence level and a one year horizon) of a position consisting of one such call option.

(10 p)