MATEMATISKA INSTITUTIONEN	Tentamensskrivning i
STOCKHOLMS UNIVERSITET	Mathematics Methods for Economist MM3001
Avd. Matematik	2021-10-25
Examinator: Sofia Tirabassi	

Instructions:

- During the exam you MAY NOT use textbooks, class notes, or any other supporting material.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear solutions even if the answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 2 Taylor polynomial of the function $f(x) = \ln(x^3 - 1)$ around the point $x_0 = 2$.

(2) Study the function $f(x) = (x^2 - 9)e^{5x}$. You should determine the following:
  (a) (3pt) places where the function is 0, local minima o maxima, where the function is concave or convex;
  (b) (2pt) sketch the graph of f. You should say if the function has limit when $x \to \pm\infty$, and, in this case what is such limit. However it is not necessary to compute in details these limits.

(3) The expression

$$x + y^3 - 2y = y^5 - x^2$$

defines $y = y(x)$ as a function of x.
  (a) (1pt) Verify that the graph $y = y(x)$ passes through the point $(1, 1)$.
  (b) (2pt) Find $y'(1)$.
  (c) (2pt) find the equation of the tangent line to the graph $y = y(x)$ in the point $(1, 1)$.

(4) Compute the following integrals:
  (a) (3pt) $\displaystyle\int \left(x^2 e^{5x^3} + \frac{20}{(7x + 1000)^{21}} + 4x \right) dx,$
  (b) (2pt) $\displaystyle\int_0^{e^3} x^2 \ln(x) dx.$

(5) Consider the matrix

$$A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & k-4 & -3 \\ -2 & k+2 & k-5 \end{pmatrix}$$

  (a) (2 pt) Compute the determinant of A, $|A|$, as a function of k.

(b) (1 pt) Find all the values of k for which A is not invertible.
(c) (2 pt) Use Gaussian elimination to solve the linear system

$$\begin{cases} x & +4y & +3z & = & 1 \\ -x & -3y & -3z & = & -1 \\ -2x & +3y & -4z & = & 3 \end{cases}$$

(6) We define the area D which contains all the $(x, y) \in \mathbb{R}^2$ which satisfy the following conditions:

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 3.$$

Let $f(x, y) = 4 + x^3 + y^3 + 3xy$.
(a) (1pt) Draw a sketch of D.
(b) (4pt) Find the maximum and minimum value of $f(x, y)$ in D.

FORMULAS

- Taylor polynomial of degree n of $f(x)$ around the point $x_0 = a$:

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

GOOD LUCK!!!

Texten på svenska

(1) (5pt) Bestäm Taylor polynomet av grad 2 till funktionen $f(x) = \ln(x^3 - 1)$ kring punkten $x_0 = 2$.

(2) Låt $f(x) = (x^2 - 9)e^{5x}$. Underök funktionen: följande ska finnas med:
  (a) (3pt) Nollställen, lokala max och minpunkter och var funktionen är konkav eller konvex;
  (b) (2p) skissa grafen av funktionen. Det ska framgå om funktionen har gränsvärden då $x \to \pm\infty$ och vilka dessa i så fall är, men gränsvärdena behöver inte motiveras utförligt.

(3) Bestäm följande integraler:
  (a) (3pt) $\int \left(x^2 e^{5x^3} + \frac{20}{(7x+1000)^{21}} + 4x \right) dx,$
  (b) (2pt) $\int_0^{e^3} x^2 \ln(x) dx.$

(4) Uttrycket

$$x + y^3 - 2y = y^5 - x^2$$

definerar $y = y(x)$ som en funktion av x.
  (a) (1pt) Se att graphen av $y = y(x)$ går genom punkten $(1, 1)$.
  (b) (2pt) Bestäm $y'(1)$.
  (c) (2pt) Hitta ekvationen till tangenten av grafen $y = y(x)$ i punkten $(1, 1)$.

(5) Låt

$$A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & k-4 & -3 \\ -2 & k+2 & k-5 \end{pmatrix}$$

  (a) (2 pt) Räkna determinanten till A, $|A|$ som en funktion av k.
  (b) (1 pt) Hitta alla värden k sådana att A inte är invertibar.
  (c) (2 pt) Använd Gausselimination för att lösa det linjär systemet nedanför

$$\begin{cases} x & +4y & +3z & = & 1 \\ -x & -3y & -3z & = & -1 \\ -2x & +3y & -4z & = & 3 \end{cases}$$

(6) Vi definearar området D som innehåler alla $(x, y) \in \mathbb{R}^2$ som upfyller att:

$$x \geq 0, \quad y \geq 0, \quad x + y \leq 3.$$

Låt $f(x, y) = 4 + x^3 + y^3 + 3xy$.
  a (1pt) Skissa området D.
  b (4pt) Bestäm max och min värde av $f(x, y)$ i D.

FORMEL

- Taylors polynom av grad n till $f(x)$ kring punkten $x_0 = a$:

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$