MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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Tentamensskrivning i
Mathematics Methods for Economist MM3001
2021-11-29

Instructions:

- During the exam you MAY NOT use textbooks, class notes, or any other supporting material.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear solutions even if the answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 2 Taylor polynomial of the function $f(x) = e^{-\sqrt{x+3}}$ around the point $x_0 = 1$.

(2) Consider the function $f(x) = \frac{x^2+x+3}{x+1}$.
   (a) (2pt) Assume that the function is defined on the interval $[0, 2]$, compute the maximum and minimum value of the function and give the points in which these values are taken.
   (b) (2pt) Assume now that the function is defined on $[0, +\infty)$. Does the function have a maximum value? Explain why.
   (c) (1pt) Compute $\lim_{x\to+\infty} \frac{f(x)}{6x}$.

(3) Consider the series $2 + \frac{4}{x} + \frac{8}{x^2} + \frac{16}{x^3} + \cdots$. Determine
   (a) (2pt) for which values of x the series converges;
   (b) (3pt) for which values of x the value of the series is 1.

(4) Compute the following integrals:
   (a) (3pt) $\int \left(x \ln(x^2+1) + 3\sqrt{x^5} \right) dx$,
   (b) (2pt) $\int \frac{5e^{2x}}{3e^{2x}-2} dx$.

(5) Consider the matrix

$$A = \begin{pmatrix} 5 & 4 & 1 \\ k-7 & k-4 & -2 \\ 5 & k-2 & 1 \end{pmatrix}$$

   (a) (2 pt) Compute the determinant of A, $|A|$ as a function of k.
   (b) (1 pt) Find all the values of k for which A is not invertible.

(c) (2 pt) Use Gauss–Jordan elimination to solve the linear system

$$\begin{cases} 5x & +4y & +1z & = & 1 \\ -7x & -4y & -2z & = & -3 \\ 5x & -2y & +z & = & 1 \end{cases}$$

(6) We define the area D to be the square in $\mathbb{R}^2$ with vertices:

$$(-1,-1), \quad (-1,1), \quad (1,-1), \quad (1,1).$$

Let $f(x,y) = xe^{3y} - e^x$.

(a) (1pt) Draw a sketch of D.

(b) (4pt) Find the maximum and minimum value of $f(x,y)$ in D.

FORMULAS

- Taylor polynomial of degree n of $f(x)$ around the point $x_0 = a$:

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$

GOOD LUCK!!!

Texten på svenska

(1) (5pt) Bestäm Taylor polynomet av grad 2 till funktionen $f(x) = e^{-\sqrt{x+3}}$ kring punkten $x_0 = 2$.

(2) Betrakta funktionen $f(x) = \frac{x^2+x+3}{x+1}$.
 (a) (2pt) Antar att f defineras på $[0, 2]$: bestäm max och min värden av f samt ge var dessa värden anta.
 (b) (2pt) Antar nu att f defineras på $[0, +\infty)$. Har funktionen nu en max värde? Förklara din mening.
 (c) (1pt) Bestäm $\lim_{x\to+\infty} \frac{f(x)}{6x}$.

(3) Bestäm följande integraler:
 (a) (3pt) $\int \left(x \ln(x^2 + 1) + 3\sqrt{x^5}\right) dx$,
 (b) (2pt) $\int_0^1 \frac{5e^{2x}}{3e^{2x} - 2} dx$.

(4) Betrakta serien $2 + \frac{4}{x} + \frac{8}{x^2} + \frac{16}{x^3} + \cdots$. Bestäm
 (a) (2pt) for vilka x convergerar serien;
 (b) (3pt) för vilka x är seriens värden 1.

(5) Låt

$$A = \begin{pmatrix} 5 & 4 & 1 \\ k-7 & k-4 & -2 \\ 5 & k-2 & 1 \end{pmatrix}$$

 (a) (2 pt) Räkna determinanten till A, $|A|$ som en funktion av k.
 (b) (1 pt) Hitta alla värden k sådana att A inte är invertibar.
 (c) (2 pt) Använd Gausselimination för att lösa det linjär systemet nedanför

$$\begin{cases} 5x & +4y & +1z & = & 1 \\ -7x & -4y & -2z & = & -3 \\ 5x & -2y & +z & = & 1 \end{cases}$$

(6) Vi definearar området D i $\mathbb{R}^2$ som kvadraten med hörnen

$$(-1, -1), \quad (-1, 1), \quad (1, -1), \quad (1, 1).$$

Låt $f(x, y) = xe^{3y} - e^x$.
 (a) (1pt) Skissa området D.
 (b) (4pt) Bestäm max och min värde av $f(x, y)$ i D.

FORMEL

- Taylors polynom av grad n till $f(x)$ kring punkten $x_0 = a$:

$$p_n(x) = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2}(x-a)^2 + \cdots + \frac{f^{(n)}(a)}{n!}(x-a)^n$$