

Algebraic Topology, VT22.  
Homework Assignment 1.  
Due Thursday January 27.

- (1) (4 points) Let  $K$  be a  $\Delta$ -complex with finitely many simplices. Define the Euler characteristic of  $K$  to be the alternating sum

$$\chi(K) = f_0 - f_1 + f_2 - \dots,$$

where  $f_n$  is the number of  $n$ -dimensional simplices in  $K$ . Find two different  $\Delta$ -structures on the torus and verify that they give the same Euler characteristic. (We will later show that the Euler characteristic is a topological invariant.)

- (2) (6 points) Compute the simplicial homology groups of the  $\Delta$ -complex obtained from two 2-simplices  $[v_0, v_1, v_2]$  and  $[w_0, w_1, w_2]$  by identifying the vertex  $v_2$  with  $w_2$  and the edge  $[v_0, v_1]$  with  $[w_0, w_1]$  using the canonical order-preserving homeomorphism.