Algebraic Topology, VT22.
Homework Assignment 2.
Due Thursday February 3.

(1) (2 points) Let $f\colon X \to Y$ be a continuous map. Assume that f admits a *section*, i.e., a continuous map $s\colon Y \to X$ such that $fs = 1_Y$. Show that $f_*\colon H_n(X) \to H_n(Y)$ is surjective for all n.

(2) (3 points) Show that $H_n(X \sqcup Y) \cong H_n(X) \oplus H_n(Y)$ for all n.

(3) (5 points) Consider the unit disk in the complex plane

$$D = \{z \in \mathbb{C} \mid |z| \leq 1\}.$$

Let X be the quotient space obtained from D by making the identification

$$z \sim z \cdot e^{\frac{2\pi i}{3}}$$

for every z on the *boundary* of D, i.e., for every $z \in D$ such that $|z| = 1$.

(a) Find a Δ-structure on X. (Hint: there exists one with 2 zero-simplices, 4 one-simplices and 3 two-simplices.)

(b) Calculate the Δ-homology of X with this structure.