Algebraic Topology, VT22.
Homework Assignment 6.
Due Thursday March 3.

(1) (5 points)
  (a) Give examples of exact sequences of abelian groups
$$A \to B \to C \to D \to E, \quad A' \to B' \to C' \to D' \to E',$$
  such that $A \cong A'$, $B \cong B'$, $D \cong D'$ and $E \cong E'$ but $C \not\cong C'$.
  (Hint: Look for examples where $A = A' = E = E' = 0$.)
  (b) Why does this not contradict the five-lemma?

(2) (5 points) Compute the homology groups of the space $\mathbb{R}^m \setminus \{x_1, \ldots, x_k\}$, where
$x_1, \ldots, x_k \in \mathbb{R}^m$ is any k-tuple of distinct points.
  (Hint: Use induction on k and apply the Mayer-Vietoris sequence to the open
subsets $A = \mathbb{R}^m \setminus \{x_1, \ldots, x_{k-1}\}$ and $B = \mathbb{R}^m \setminus \{x_k\}$ of $\mathbb{R}^m$.)