Algebraic Topology, VT22.
Homework Assignment 7.
Due Thursday March 10.

(1) (5 points) Compute the degrees of the following self-maps of S^n:
  (a) Reflection in a line through the origin.
  (b) Rotation by the angle π in a coordinate plane.
  (c) The map $(x_0, \ldots, x_n) \mapsto (x_{\omega_0}, \ldots, x_{\omega_n})$, where ω is a permutation.
  (d) (Bonus problem) The map $x \mapsto Ax$, where A is an orthogonal $(n+1) \times (n+1)$-matrix.

(2) (5 points) Let X be cell complex.
  (a) Following Hatcher's proof of Theorem 2.35, show that an isomorphism

$$\phi_X \colon H_n(X) \to H_n^{CW}(X)$$

is given explicitly by

$$\phi_X(x) = [j_n(y)],$$

where $y \in H_n(X^n)$ is any homology class that maps to x under the homomorphism $H_n(X^n) \to H_n(X)$ and where $j_n \colon H_n(X^n) \to H_n(X^n, X^{n-1})$ is the canonical homomorphism.
  (b) A map $f \colon X \to Y$ between cell complexes is called cellular if $f(X^n) \subseteq Y^n$ for all n. Show that a cellular map induces a homomorphism

$$f_*^{CW} \colon H_n^{CW}(X) \to H_n^{CW}(Y)$$

for all n, and show that the diagram

$$\begin{array}{ccc} H_n^{CW}(X) & \xrightarrow{f_*^{CW}} & H_n^{CW}(Y) \\ \downarrow{\scriptstyle \phi_X} & & \downarrow{\scriptstyle \phi_Y} \\ H_n(X) & \xrightarrow{f_*} & H_n(Y) \end{array}$$

is commutative.