

Algebraic Topology,
Homework Assignment 8,
Due Thursday March 24, 2022

(1) (5 points)

Let a, b, c, d be the edges of a square, oriented counterclockwise. The Klein bottle K is given by identifying the edge a with the edge c and the edge b with the edge \bar{d} , i.e. the edge d with the reversed orientation.

- (a) Find a CW structure on K with as few cells as possible.
- (b) Determine the cellular chain complex for your CW structure and compute its homology.
- (c) Use the universal coefficient theorem to compute $H^*(K; \mathbf{Z})$.
- (d) Verify the answer by computing $H^*(K; \mathbf{Z})$ directly from your cellular chain complex.

(2) (5 points)

Let's call an abelian group A *liberal* if the functor $B \mapsto \text{Hom}(A, B)$ is exact. Show that A is liberal if and only if A is free.