Algebraic Topology,
Homework Assignment 8,
Due Thursday March 31, 2022

(1) (5 points)
Derive the following theorem from the Borsuk–Ulam theorem: if $\mathbf{S}^n$ is covered by $n+1$ compact sets $A_1, \ldots, A_{n+1}$ then one of these sets must contain a pair of antipodal points (i.e. x and $-x$). *Hint:* Suppose not, and construct a map $f \colon \mathbf{S}^n \to \mathbf{R}^n$ such that $f_i(x) = 0$ if $x \in A_i$ and $f_i(x) = 1$ if $-x \in A_i$ for $1 \leq i \leq n$.

(2) (5 points)
   (a) Show that the cup product is well-defined on relative cohomology groups:
   $$H^*(X, A) \otimes H^*(X, B) \xrightarrow{\cup} H^*(X, A \cup B).$$
   (b) Show that if X can be covered with n contractible open sets U_i, then all n-fold cup products in $\tilde{H}^*(X)$ vanish.
   (c) Conclude that the cup product is always zero in $\tilde{H}^*(\Sigma X)$, where ΣX denotes the suspension of the space X.