Algebraic Topology,
Homework Assignment 11,
Due Thursday April 21, 2022

(1) (5 points)
Let M be a manifold of dimension $n > 0$ and for $k \geq 2$, consider the space $\binom{M}{k}$ of k-element subsets of M, topologized as a subset of M^k/Σ_k, where the symmetric group Σ_k acts by permutations of the factors. Show that $\binom{M}{k}$ is a manifold, but M^k/Σ_k is not.

(2) (5 points; generalizing Hatcher Ex. 3.3.2)
Let M be a connected manifold of dimension n. A k-dimensional submanifold of M is a subspace N such that for each $x \in N$ there exists a neighborhood U in M, containing x, and a homeomorphism $\phi\colon U \to V$ onto an open subspace V of $\mathbf{R}^n$ in such a way that $\phi^{-1}(V \cap \mathbf{R}^k)$ is an open neighborhood of x in N.

Let N be a submanifold of codimensions greater than one (i.e. dimension less than $n-1$). Show:
(a) $M - N$ is a connected manifold.
(b) M is orientable if and only if $M - N$ is orientable.