MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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Written examination in
Mathematics III – ODE
1 June 2021

1. **(4p)** Solve the initial value problem $y' = xy^2 + x$, $y(0) = 1$.

2. **(6p)** Let $a \in \{1, \ldots, 12\}$ be the number of your month of birth. (For instance, $a = 1$ if you are born in January, $a = 7$ if you are born in July, or $a = 10$ if you are born in October.) For your a, determine the general solution to the system

$$\begin{cases} x' = -x + y, \\ y' = -x - 3y, \\ z' = -x - (a+3)y + az. \end{cases}$$

3. **(4p)** Consider the initial value problem

$$y' = 2x + y + 1, \quad y(1) = 1.$$

Compute a numerical solution at $x = 3$ by using the (forward) Euler method with step length $h = 1/2$.

4. **(4p)** Show that for each $x_0 \in \mathbb{R}$ the initial value problem

$$y' = \frac{3|y| \cos x}{2 + x^2}, \quad y(x_0) = 0,$$

has a unique solution defined on all of $\mathbb{R}$.

5. **(6p)** Let again a be the number of your month of birth. Determine all equilibrium points of the autonomous system

$$\begin{cases} \frac{dx}{dt} = x^2 - y^2, \\ \frac{dy}{dt} = (-1)^a (a+1)x + y - 1, \end{cases}$$

and investigate whether these equilibrium points are asymptotically stable.

6. **(6p)** Let again a be the number of your month of birth. Consider the boundary value problem

$$2y'' - ay' = f(x) \text{ on } [0, 1], \qquad y(0) = c_0, \quad y(1) = c_1. \tag{1}$$

(a) Prove that for each $f \in \mathcal{C}[0, 1]$ and all $c_0, c_1 \in \mathbb{R}$ the problem (1) has a unique solution.

(b) Solve the problem (1) for $f(x) = e^{ax}$, $c_0 = 0$ and $c_1 = e^a/a^2$.