1. **(4p)** Solve the initial value problem $\frac{\mathrm{d}u}{\mathrm{d}t} = e^u \sin t$, $u(0) = 0$.

2. **(6p)** Let $a \in \{1, \ldots, 12\}$ be the number of your month of birth. (For instance, $a = 1$ if you are born in January, $a = 7$ if you are born in July, or $a = 10$ if you are born in October.) For your a, determine the general solution to the system

$$\begin{cases} x' = -(a+2)x + (2a+2)y, \\ y' = -(a+1)x + (2a+1)y, \\ z' = (2a+2)x - (2a+2)y + az. \end{cases}$$

3. **(4p)** Determine all functions $y : \mathbb{R} \to \mathbb{R}$ such that y' and xy are linearly dependent.

4. **(5p)** Use the **Laplace transform** to solve the initial value problem

$$u'' + 4u' + 3u = 12, \quad u(0) = 7, \quad u'(0) = 1.$$

5. **(6p)** Let again a be the number of your month of birth. Rewrite the differential equation

$$x'' + ax' - x^2 + 1 = 0$$

as a first-order system and compute all its equilibrium points. Moreover, for each equilibrium point, investigate whether it is unstable/stable/asymptotically stable.

6. **(5p)** Determine the general solution to the differential equation

$$y'' - y = xe^x.$$