Algebraic Topology,
Homework Assignment 12
Due Thursday April 28, 2022

(1) (5 points)
Let $p\colon Y \to X$ be a covering space and $K \subset X$ a compact
subset. Show:

(a) For every section $s\colon K \to Y$ (i.e. continuous function such
that $p \circ s = \mathrm{id}_K$), there exists an open set $U \supset K$ and an
extension of s to U.

(b) Let $U \supset K$ be open and $s, s'\colon U \to Y$ be two sections that
agree on K. Then there is a possibly smaller open set V,
$U \supset V \supset K$, such that $s|_V = s'|_V$.

(2) (5 points)
Let M, N be connected, compact, oriented manifolds of the
same dimension n. Then the mapping degree $\deg(f)$ of a map
$f\colon M \to N$ is defined to be the integer k such that the homo-
morphism

$$\mathbf{Z} \xrightarrow{o_M^{-1}} H_n(M) \xrightarrow{f_*} H_n(N) \xrightarrow{o_N} \mathbf{Z}$$

is multiplication by k. Here the isomorphisms o_M and o_N are
determined by the orientations.

Show: a map of nonzero mapping degree is surjective.