Logic Exam 22-05-31 Solution

Exercise 1

This is a theory question we refer to the book/notes

Exercise 2

(a)

	v
P_1	0
P_2	1
P_3	0

(b)

	v_1	v_2
P_1	1	1
P_2	0	1
P_3	1	0

Exercise 3

(a) $[\![P_1(x_1 x_2) \longrightarrow P_1(x_2 x_1)]\!]^{R v} = 1$ iff

$v(x_1) \geq v(x_2)$

or

$v(x_1) < v(x_2)$ and $v(x_2) < v(x_1)$

iff

$0 \geq 0$ OR $(0<0$ and $0<0)$

as $0 \geq 0$ is true we have that

$[\![P_1(x_1 x_2) \longrightarrow P_1(x_2 x_1)]\!]^{R v} = 1$

(b) $[\![\forall x_2 \; P(x_1 x_2) \longrightarrow P_1(x_2 x_1)]\!]^{R v} = 1$ iff for every real number a

$$[\![P_1(x_1, x_2) \longrightarrow P_1(x_2\, x_1)]\!]^{R\; v[x_2 \to a]} = 1$$

This is equivalent to

$v(x_1) \geq a$

or

$v(x_1) < a$ and $v(x_1) > a$

If we take $a = 1$ we have that

$v(x_1) = 0 < a$ but 0 is not ≥ 1

So this is false

$$[\![\forall x_2 P(x_1, x_2) \longrightarrow P(x_2\, x_1)]\!]^{R, v} = 0$$

Exercise 4

(a) Consider the following model

	v
P_1	0
P_2	1
P_3	0

This is a countermodel for the entailment.

(b) $\vee E_2$ is not used correctly

$$\dfrac{A \vee B \qquad \begin{array}{c} A \\ \vdots \\ C \end{array} \qquad \begin{array}{c} B \\ \vdots \\ C \end{array}}{C}$$

They are independent.

Exercise 5

Preliminary consideration

$$FV(\neg P_1(x_3\, x_2\, x_4\, x_5)) = \{x_3\, x_2\, x_4\, x_5\}$$

(a) Any term is free for x_2, x_3 in φ

(b) $t = x_2$ is not free for x_4 in φ.

Exercise 6

(a) x_1 is a free variable in $P_2(x_1)$ so $\exists E^2$ is inconnect

(b)

$$\dfrac{\dfrac{[\exists x_1 P_1(x_1)]^2 \qquad \dfrac{\dfrac{\dfrac{\forall x_1\, P_1(x_1) \to P_2(x_1)}{P_1(x_1) \to P_2(x_1)}\,\forall E \qquad [P_1(x_1)]^1}{P_2(x_1)}\,\to E}{\exists x_1 P_2(x_1)}\,\exists I}{\exists x_1 P_2(x_1)}\,\exists E_1}{\exists x_1 P_1(x_1) \to \exists x_1 P_2(x_1)}\,\to I, 2$$

(a) $\forall x_1 x_2 \; P_1(x, x_2)$

Exercise 7

(a) Let

$\Gamma_n' = \{\neg P_1 \dots \neg P_n\}$ this has a model

$[\![P_i]\!]^v = 0$ for $i = 1 \dots n$

So is consistent by soundness

Then $\Gamma' = \bigcup_n \Gamma_n'$ is consistent.

(b) This is not an element. in any model for Γ^* we have that $[\![P_3]\!]^v = [\![P_2]\!]^v = 0$

$[\![P_2 \to P_3]\!]^v = 1$

$[\![P_2]\!] = 0$

So $[\![(P_2 \to P_3) \to P_2]\!]^v = 0$.

Exercise 8

$$\dfrac{\dfrac{\neg\forall x_1(\varphi\wedge\psi) \qquad \dfrac{\dfrac{\dfrac{\dfrac{[\neg\exists x_1(\neg\varphi\vee\neg\psi)] \quad \dfrac{\dfrac{[\neg\varphi]^1}{\neg\varphi\vee\neg\psi}\vee I}{\exists x_1(\neg\varphi\vee\neg\psi)}\exists I}{\bot}\to E}{\varphi}RAA^1 \qquad \dfrac{\dfrac{[\neg\exists x_1 \neg\varphi\vee\neg\psi] \quad \dfrac{\dfrac{[\neg\psi]^2}{\neg\varphi\vee\neg\psi}\vee I \; 3}{\exists x_1 \neg\varphi\vee\neg\psi}\exists I}{\bot}\to E}{\psi}RAA^2}{\varphi\wedge\psi}\wedge I}{\forall x_1 \; \varphi\wedge\psi}\forall I \;\; x_1 \text{ not free in UA}}{\bot}}{\exists x_1(\neg\varphi\vee\neg\psi)}RAA_2$$

Exercise 9

(a) $\forall x_1 x_2 \; P_1(x_1 x_2) \to \exists x_3 \; P(x_1 x_3) \wedge P(x_3 x_2)$.

(b) $\exists x_0 \; \forall x x_2 x_3 \; P_1(x_0 x) \wedge P_1(x_2 x_3) \to P_1(f_2(x_2 x), f_2(x_3 x))$

(c) $\exists x_0 \; \forall x_1 \; P_1(x_0, f_1(x_1 x))$

(d) $([0,1]; \leq; \times)$ The formula is true

this is $\{a^2 / a \in [0,1]\}$ is bounded above and below

$(\mathbb{R}; \leq; \times)$ the formula is false

(These are examples, other structure apply