STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2022-05-24

Examiner: Kristoffer Lindensjö; kristoffer.lindensjo@math.su.se

Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

(A) Find the value of a bond with face value $F = 100$, coupons $C = 10$ which are paid every six months, and maturity in 1.5 years. The interest rate (which is constant and **continuously compounded**) is 3%. (5p)

(B) Find the value of a bond with no face value ($F = 0$), annual coupons $C = 100$, and maturity in 30 years. The interest rate (which is constant and **compounded yearly**) is 5%. (5p)

Problem 2

Suppose we have a one-period binomial model. The risk-free interest rate (one-period return on the risk-free asset) is R. The current share price is $S(0)$ and the share price at time 1 will be either $S^u = S(0)(1+U)$ or $S^d = S(0)(1+D)$. Suppose we have a derivative with maturity at time 1 and payoff function

$$f(s) = \sqrt{s}$$

and that

$$R = 0.05, \quad U = 0.1, \quad D = -0.1, \quad S(0) = 1.$$

(A) Calculate the value of the derivative in case it is European. (7 p)

(B) Calculate the value of the derivative in case it is American. (3 p)

Problem 3

Consider the following three bonds:

- a one-year zero-coupon bond with face value 100 SEK, trading at 93 SEK,
- a two-year coupon bond with annual coupons of 10 SEK and face value 100 SEK, trading at 103 SEK,
- a three-year coupon bond with annual coupons of 7 SEK and face value 100 SEK, trading at 99 SEK.

(A) Derive the spot rates. (5 p)

(B) If the term structure is deterministic, what is the price at time $t = 1$ of a zero-coupon unit-bond maturing at time $T = 3$? (5 p)

Problem 4

Consider a market consisting of three stocks, with expected returns 0.22, 0.17 and 0.07, and variances of returns 0.03, 0.02 and 0.01 respectively. The covariance between the returns of the first and second stock is 0.01. The return of the third stock is uncorrelated to the first and second stock. The risk-free return is 0.02.

(A) Calculate the weights of the portfolio consisting of the two stocks that has the smallest variance. (3 p)
Hint: you may use the following

$$\mathbf{w}_{\text{MVP}} = \frac{\mathbf{u}\mathbf{C}^{-1}}{\mathbf{u}\mathbf{C}^{-1}\mathbf{u}^{\top}},$$

where $\mathbf{u}$ *is a vector of ones, and* $\mathbf{C}$ *is the covariance matrix of the returns of the stocks, and*

$$\mathbf{C}^{-1} = \begin{pmatrix} 40 & -20 & 0 \\ -20 & 60 & 0 \\ 0 & 0 & 100 \end{pmatrix}.$$

(B) What is the variance of the portfolio in (B)? (2 p)

(C) Calculate the weights of the market portfolio. (2 p)
Hint: you may use the following

$$\mathbf{w}_M = \frac{(\mathbf{m} - R\mathbf{u})\mathbf{C}^{-1}}{(\mathbf{m} - R\mathbf{u})\mathbf{C}^{-1}\mathbf{u}^{\top}},$$

where $\mathbf{m}$ *is the mean vector of the returns of the stocks, and* R *is the risk-free return.*

(D) Does the portfolio with weights $\mathbf{w}_V = (0.25, 0.30, 0.45)$ lie on the *efficient frontier*? (3 p)
Hint: You may use the following: Each portfolio on the minimum variance line can be obtained as a linear combination of any two portfolios on the minimum variance line with different expected returns.

Problem 5

Consider a European put and an American put on the same underlying share with price process $S(t)$, with the same strike X and the same maturity date T. Suppose there are no dividends etc. The interest rate is constant $r > 0$. The current time is $t = 0$. Let $P_A(0)$ be the value of the American put and $P_E(0)$ be the value of the European put. Show using the no-arbitrage principle that

$$P_E(0) \leq P_A(0).$$

(10 p)

Hint: use the usual proof by contradiction approach.