STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2022-08-17

Examiner: Kristoffer Lindensjö; kristoffer.lindensjo@math.su.se

Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

The current time is $t = 0$ is and the continuously compounded interest rate is $r = 3\%$.

(A) Consider a bond that matures in 2 years with face value $F = 100$ and annual coupons $C = 10$. What is the value of the bond at $t = 0$? (4 p)

(B) Consider a zero coupon bond with face value 1 that matures in 5 years. What is the value of the bond at $t = 0$? How many years after $t = 0$ will the bond be worth 0.9? (6 p)

Problem 2

Consider the following two bonds:

- a one-year zero-coupon bond with face value 100 SEK, trading at 95 SEK,
- a two-year zero-coupon bond with face value 100 SEK, trading at 92 SEK

(A) Derive the spot rates. (3 p)

(B) If the term structure is deterministic, at what rate could you borrow money between $t = 1$ and $t = 2$? (4 p)

(C) You want to take a loan of 10 000 SEK one year from now, and repay the loan after another year. If the term structure is stochastic, explain in detail how you can arrange this loan today at the fixed interest rate determined in (B), by investing in (buying or selling/shorting) the two zero-coupon bonds. (3 p)

Problem 3

Consider a market consisting of two stocks, with expected returns 0.1, and 0.2, and standard deviations of returns 0.16 and 0.25 respectively. The correlation between returns of the two stocks is 0.4. Short selling is allowed. The risk-free return is 0.03.

(A) Calculate the weights of the portfolio consisting of the two stocks that has the smallest variance. (4 p)
Hint: you may use the following

$$\mathbf{w}_{\text{MVP}} = \frac{\mathbf{u}\mathbf{C}^{-1}}{\mathbf{u}\mathbf{C}^{-1}\mathbf{u}^{\top}}, \tag{1}$$

where $\mathbf{u}$ *is a vector of ones, and* $\mathbf{C}$ *is the covariance matrix of the returns of the stocks.*

(B) State the optimization problem that (1) is the solution to (for the general case, when we have a market consisting of n risky assets). (3 p)

(C) Show that (1) is the solution to the optimization problem in (B) (assuming that $\det \mathbf{C} \neq 0$). (3 p)

Problem 4

The value of 1 EUR is currently equal to that of 1 USD. The annual interest rate for borrowing/lending in EUR is 1% (yearly compounding). The annual interest rate for borrowing/lending in USD is 3% (yearly compounding).

Find the forward price in EUR for delivery of 100 USD in two years.

(10 p)

Problem 5

Suppose you are a Swedish investor and that you want to price a European call option on 1 EUR in terms of the Swedish currency SEK. The current ($t = 0$) exchange rate is $S(0) = 10$ and the strike price of the option is also $X = 10$.

Your choice of model is a one-period binomial model and you believe that the exchange rate will at time 1 (when the option expires) be either $S^u = S(0)(1+U)$ or $S^d = S(0)(1 + D)$, where $U = 0.1$ and $D = -0.1$.

The return on a risk-free asset in SEK is $K_{SEK} = 1\%$ and the return on a risk-free asset in EUR is $K_{EUR} = 2\%$.

What is the current value of the call option? (10 p)

Hint: view 1 EUR as the underlying asset.