

**Instructions:**

- During the exam you MAY NOT use textbooks, class notes, or any other supporting material.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- You can use the formula sheet that come with the exam.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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- (1) (5pt) Compute the degree 3 Taylor polynomial of the function  $f(x) = xe^{x^2}$ , around the point  $x_0 = 0$ , and use it to give an approximation of  $f(0.1)$ .
- (2) Geometric Series:
  - (a) (3 pt) Determine for which  $x$  the following series converges:
$$S(x) = 5 - 10e^{-x} + 20e^{-2x} - 40e^{-3x} + \dots$$
  - (b) (2pt) Determine if there is a  $x$  such that  $S(x) = \frac{1}{3}$ .
- (3) Consider the function  $f(x) = (2x + 1)e^{-x^2+1}$ .
  - (a) (2pt) Find all the critical points and determine their type.
  - (b) (1pt) Find where the function is increasing or decreasing.
  - (c) (1pt) Find the max and min value of the function on the interval  $[-1, 2]$ .
  - (d) (1pt) Compute  $\lim_{x \rightarrow \pm\infty} f(x)$  and sketch the graph of  $f$ .
- (4) Compute the following integrals:
  - (a) (3pt)  $\int (\sqrt{t}e^{\sqrt{t}} + \sqrt[5]{t^3}) dt$ ,
  - (b) (2pt)  $\int_0^1 \frac{3y}{y^2 + 1} dy$ .
- (5) Consider the matrix
$$A = \begin{pmatrix} 2 & 0 & 2+c \\ 3 & -1 & 0 \\ c & 0 & -2 \end{pmatrix}$$
  - (a) (2 pt) Compute the determinant of  $A$ ,  $|A|$  as a function of  $c$ .
  - (b) (1 pt) Find all the values of  $c$  for which  $A$  is not invertible.

(c) (2 pt) Find the solution of the following linear system:

$$\begin{cases} 2x & & -2z & = & 4 \\ 3x & -1y & +3z & = & 13 \\ -2x & & -2z & = & -8 \end{cases}$$

(6) Consider the two variables function

$$f(x, y) = e^{xy-x-y}$$

defined on the triangle

$$D = \{(x, y) \mid x \geq 0, y \geq 0, y \leq 4 - x\}$$

- (a) (2pt) Find all the critical points of  $f(x, y)$  - even those lying outside  $D$  and determine their type.
- (b) (2pt) Determine candidates for the maximum and minimum points of  $f$  on  $D$  that lie on *boundary* of  $D$ . (In order to get credit you have to explain what you are doing, the correct answer without the right explanation will not be accepted)
- (c) (1 pt) Determine the minimum and the maximum value of  $f(x, y)$  on  $D$ . (In order to get credit you have to explain what you are doing, the correct answer without the right explanation will not be accepted)

GOOD LUCK!!!

**Senska texten, (formular finns ovanför)**

- (1) (5pt) Beräkna grad 3 Taylor polinom till funktioner  $f(x) = xe^{x^2}$ , omkring punkten  $x_0 = 0$ , och använd det för approximera  $f(0.1)$ .

- (2) Geometrisk Serier:

- (a) (3 pt) Bestäm för vilka  $x$  den följande konvergerar:

$$S(x) = 5 - 10e^{-x} + 20e^{-2x} - 40e^{-3x} + \dots$$

- (b) (2pt) Bestäm om det finns  $x$  sådan att  $S(x) = \frac{1}{3}$ .

- (3) Betrakta funktionen  $f(x) = (2x + 1)e^{-x^2+1}$ .

- (a) (2pt) Hitta alla de kritiska punkterna och bestäm dess typ.

- (b) (1pt) Bestäm var funktionen är växande och avtagande.

- (c) (1pt) Hitta den största och den minsta värden till funktionen i  $[-1, 2]$ .

- (d) (1pt) Räkna  $\lim_{x \rightarrow \pm\infty} f(x)$  och skissa grafen till  $f$ .

- (4) Räkna de följande integralerna:

(a) (3pt)  $\int \left( \sqrt{t}e^{\sqrt{t}} + \sqrt[5]{t^3} \right) dt,$

(b) (2pt)  $\int_0^1 \frac{3y}{y^2 + 1} dy.$

- (5) Betrakta matrisen

$$A = \begin{pmatrix} 2 & 0 & 2+c \\ 3 & -1 & 0 \\ c & 0 & -2 \end{pmatrix}$$

- (a) (2 pt) Räkna determinanter till  $A$ ,  $|A|$  som en funktion av  $c$ .

- (b) (1 pt) Hitta alla värden  $c$  sådan att  $A$  inte är invertibär.

- (c) (2 pt) Räkna lösningen till system

$$\begin{cases} 2x & -2z & = & 4 \\ 3x & -1y & +3z & = & 13 \\ -2x & & -2z & = & -8 \end{cases}$$

- (6) Betrakta den följande funktionen av två variabler

$$f(x, y) = e^{xy-x-y}$$

som defineras i trekanten

$$D = \{(x, y) \mid x \geq 0, y \geq 0, y \leq 4 - x\}$$

- (a) (2pt) Hitta alla kritiska punkter till  $f(x, y)$  - punkter som ligger utanför  $D$  också behövs att hitta.

- (b) (2pt) Hitta kandidater för största och den minsta punkter till  $f$  på  $D$  som ligger på *gränsen* av  $D$ .

- (c) (1 pt) Beräkna den största och den minst värden till  $f$  på på  $D$ .

LYCKA TILL!!!