MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
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MM5022 HT 2022
Tentamen 24 oktober 2022

No calculators, notes or computers are allowed. A valid solution to one of the problems below gives 5 points. Approximately half the total points (if roughly evenly distributed) will suffice for pass. Do not forget to state explicitly the results from the course that you use, and to give the details of your arguments.

EXAM

1. Determine all entire functions f for which (i) $f(1) = f'(1) = 0$ and (ii) $|f(z)| \leq vertz - 1|^2$ for all $z \in \mathbb{C}$.

2. Describe the region in the w-plane that is the image of
$$\{z \in \mathbb{C} \ : |z| < 1, \ \text{Re}z > 0, \ \text{Im}z > 0\},$$
under the mapping
$$w = \frac{z^2 + 1}{z^2 - 1}.$$

3. Determine the residue for each pole of the function
$$f(z) = \frac{2 + z^7}{z^4(z + 1)^3}.$$
Use the result to calculate
$$\int_C f(z)dz,$$
where C is the curve $|\text{Re } z| + |\text{Im } z| = 0.5$, oriented counterclockwise.

4. Calculate the integral
$$\int_{-\infty}^{\infty} \frac{\sin \pi x}{x(1 - x^2)} dx.$$

5. Determine the Laurent series of the function
$$\frac{z^2}{z^2 - 4z + 3},$$
in the annulus $1 < |z| < 3$.

6. (a) Show that a function $f(z)$, which is analytic in a neighbourhood of $z = 0$, may be developed in a series

$$f(z) = \sum_{n=0}^{\infty} a_n z^n (1+z)^{-n}, .$$

where the series is convergent and the equality is true in some neighbourhood of $z = 0$.

(b) Determine the first three non-zero coefficients in such a series expression for $\sin z$.