MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sofia Tirabassi

Tentamensskrivning i
Mathematics Methods for Economist MM3001
2022-12-12

Instructions:

- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 3 Taylor polynomial of the function $f(x) = (x+1)\ln(x^2+1)$, around the point $x_0 = 0$, and use it to give an approximation of $f(0.1)$.

(2) The equation

$$3x^2y + e^{x+y} + \ln(x+y) - e^y = 0$$

defines y as a function of x, that is $y = y(x)$.
(a) (2 pt) Find the value of y when $x = 0$.
(b) (2 pt) Compute the slope of the tangent line to the curve in $(0, 1)$.
(c) (1pt) Give the equation of the tangent line to the curve in $(0, 1)$.

(3) Consider the function $f(x) = \frac{x^2+9}{x}$.
(a) (2pt) Find all the critical points and determine their type.
(b) (2pt) Find where the function is increasing or decreasing and concave or convex.
(c) (1pt) Find the max and min value of the function on the interval $[1, 4]$.

(4) Compute the following integrals:
(a) (3pt) $\int \left(\frac{3}{\sqrt{t}} \ln(\sqrt{t}) + \frac{3}{t+1} \right) dt$,
(b) (2pt) $\int_0^{+\infty} ye^{-y^2+1} dy$.

(5) Consider the matrix

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 40 & c \\ 1 & c & 6 \end{pmatrix}$$

(a) (2 pt) Compute the determinant of A, $|A|$ as a function of c.

(b) (1 pt) Find all the values of c for which A is not invertible.
(c) (2 pt) Determine whether the following linear system has 1, 0, or infinitely many solutions:

$$\begin{cases} x & +2y & & = & 1 \\ 2x & +4y & & = & 2 \\ x & & +6z & = & 1 \end{cases}$$

(6) Consider the two variables function

$$f(x,y) = 5 - 3y + 3x^2y + 2(x^3 + y^3)$$

(a) (2pt) Find all the critical points of $f(x,y)$ and determine their type.
(b) (2pt) Consider now D, the square with vertices $(0,0)$, $(1,0)$, $(1,-1)$, and $(0,-1)$. Determine the candidates for the the maximum and minimum value of f on D which lies on the *boundary* of D.
(c) (1 pt) Determine the minimum and the maximum value of $f(x,y)$ on D.

GOOD LUCK!!!

Svenska Texten

(1) (5pt) Berk̈na grad 3 Taylor polynom till funtionen $f(x) = (x+1)\ln(x^2+1)$, kring $x_0 = 0$, och använda det för att approximera $f(0.1)$.

(2) Ekvationen
$$3x^2y + e^{x+y} + \ln(x+y) - e^y = 0$$
definerar y som en funktion av x, i. e. $y = y(x)$.
(a) (2 pt) Hitta vŕden för y när $x = 0$.
(b) (2 pt) Beräkan lutningen till den tangenta linjen till kurvan i $(0, 1)$.
(c) (1pt) Ger ekvationen till den tangenta linjen til kurvan i $(0, 1)$.

(3) Betrakar funktionen $f(x) = \frac{x^2+9}{x}$.
(a) (2pt) Hitta alla de kritiska punkterna och bestäm deras typ
(b) (2pt) Bestäm var funktionen är vaxande eller evtagende.
(c) (1pt) Bestäm de max och min värderna till funktionen i intervallen $[1, 4]$.

(4) Beräkna följande integraler:
(a) (3pt) $\int \left(\frac{3}{\sqrt{t}} \ln(\sqrt{t}) + \frac{3}{t+1} \right) dt$,
(b) (2pt) $\int_0^{+\infty} ye^{-y^2+1} dy$.

(5) Betrakta matrisen
$$A = \begin{pmatrix} 1 & 2 & 0 \\ 2 & 40 & c \\ 1 & c & 6 \end{pmatrix}$$
(a) (2 pt) Beräkna determinanten till A, $|A|$ som en funktion av c.
(b) (1 pt) Hitta alla värderna tillc c var A inteär invertibara.
(c) (2 pt) Best"am om fóljande system har 0, 1 och oendliga många lössningar:
$$\begin{cases} x & +2y & & = & 1 \\ 2x & +4y & & = & 2 \\ x & & +6z & = & 1 \end{cases}$$

(6) Betrakta följande funktionen i två varibler:
$$f(x, y) = 5 - 3y + 3x^2y + 2(x^3 + y^3)$$
(a) (2pt) Hitta alla kritiska punkterna för $f(x, y)$ och bestäm deras typ;
(b) (2pt) Btrakta nu D, kvadraten med hjörn $(0, 0)$, $(1, 0)$, $(1, -1)$, och $(0, -1)$. Betrakta kandidater för max och min värden av f på D som ligger på grensen av D.
(c) (1 pt) Bestäm de max och min värden till$f(x, y)$ på D.

Lycka Till!!!