MATEMATISKA INSTITUTIONEN
STOCKHOLM UNIVERSITET
Avd. Matematik
Examinator: Yishao Zhou

Tentamensskrivning i
Dynamiska system och
optimal kontrollteori, AN
den 25 oktober 2019

Open book exam: Sontag, Mathematical Control Theory, Deterministic finite dimensional systems, 2nd edition, Springer. No other material is allowed.
There are 5 problems with 10 points each. They are not arranged in degree of difficulty.

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(1) Consider the linear time invariant system $\dot{x}(t) = Ax(t) + Bu(t)$ starting at $x(0) = 0$, where $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Let $\mathcal{R}_t := \{\xi \in \mathbb{R}^n : \exists u \text{ such that } x(t) = \xi\}$. Show that, for all $t > 0$, $\mathcal{R}_t = \mathcal{R}(A, B)$.

(2) Is the following system stabilizable?

$$\frac{dx}{dt} = \begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} u$$

Determine the control law $u = Kx$ so that the system has its poles at $-2, -2, -1, -1$, if the answer is affirmative. Otherwise show that it is impossible. Is it possible to place all poles at -2? Justify your answer.

(3) Assume that $(A, C) \in \mathcal{S}^{\text{obs}}_{n,p}$. Show that every symmetric solution, K, of the following matrix equation

$$-A'K - KA + KLK - C'C = 0$$

is nonsingular.

(4) Consider the nonlinear system

$$\dot{x}_1 = -x_1 + g(x_2)$$
$$\dot{x}_2 = -x_2 + h(x_1)$$

where $g(0) = h(0) = 0$, and $|g(u)| \le |u|/2$ and $|h(u)| \le |u|/2$. Show that it is globally asymptotically stable.

(5) Let $A = \begin{pmatrix} -\frac{1}{g_1 g_2} & -\frac{1}{\sqrt{g_3}\, g_2 \sqrt{g_1}} & 0 \\ -\frac{1}{\sqrt{g_1}\, g_2 \sqrt{g_3}} & -\frac{1}{g_3 g_2} - \frac{1}{g_3 g_4} & -\frac{1}{\sqrt{g_5}\, g_4 \sqrt{g_3}} \\ 0 & -\frac{1}{\sqrt{g_3}\, g_4 \sqrt{g_5}} & -\frac{1}{g_5 g_6} - \frac{1}{g_5 g_4} \end{pmatrix}$, $b = \begin{pmatrix} \frac{1}{\sqrt{g_1}} \\ 0 \\ 0 \end{pmatrix} = c$. Show that

$$c'(sI - A)^{-1}b = \cfrac{1}{g_1 s + \cfrac{1}{g_2 + \cfrac{1}{g_3 s + \cfrac{1}{g_4 + \cfrac{1}{g_5 s + \frac{1}{g_6}}}}}}$$

and assume that $g_i > 0$.

Is this realization minimal?

Determine a minimal realization for the sequence

$\mathcal{A} = \{1, -1, 2, -5, 14, -42, 131, -417, 1341, -4332, 14041, ...\}$. Can $\mathcal{A}$ be realized in the above form? In this case determine g_i's.

Skrivningsåterlämning äger rum i rum 403, hus 6 den 8 november 2019 kl 15:00, därefter på studentexpedition.