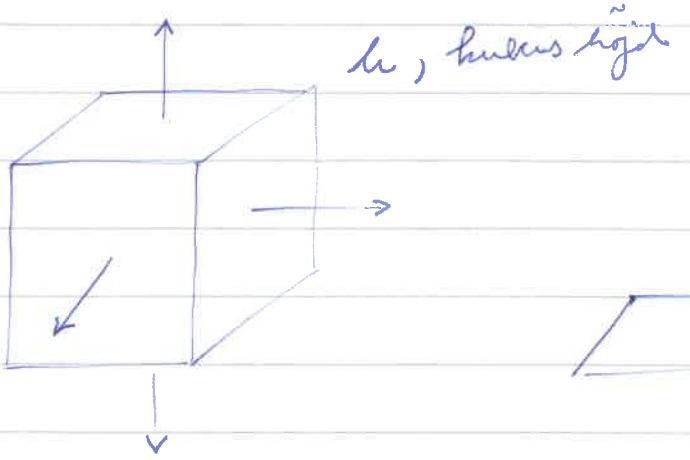
Gauss sats: Bevisskiss för en kub.



* Dela upp $\vec{n}$: $n_1(1,0,0) + n_2(0,1,0) + n_3(0,0,1)$.

* Hantera $n_3(0,0,1) = (0,0,n_3)$.

Vi har $\operatorname{div}(0,0,n_3) = \frac{\partial n_3}{\partial z}$.

Nu:

$$\iiint_K \frac{\partial n_3}{\partial z}\,dx\,dy\,dz$$

$$= \iint_B \left\{ \int_0^h \frac{\partial n_3}{\partial z}\,dz \right\} dx\,dy$$

$$= \iint_B \Big[n_3(x,y,z) \Big]_0^h dx\,dy$$

A.J.

$$= \iint_B \left(u_3(x,y,h) - u(x,y,0) \right) dx\,dy$$

$$= \iint_B u_3(x,y,h)\, dx\,dy + \iint_B -u_3(x,y,0)\, dx\,dy$$

$$= \iint_B u_3(x,y,u)\,(0,0,1)\cdot(0,0,1)\, dx\,dy + \iint_B u_3(x,y,0)\,(0,0,1)\cdot(0,0,-1)\, dx\,dy$$

$$= \iint_{\text{Lehyta}} (0,0,u_3)\cdot \vec{N}\, dS + \iint_{\text{Bottenyta}} (0,0,u_3)\cdot \vec{N}\, dS$$

$$\underbrace{+0+0+0+0}_{(*)}$$

$$= \iint_{\partial K} (0,0,u_3)\cdot \vec{N}\, dS.$$

$\not\equiv$

A.S.

Sidorna ger bidrag 0:

$\vec{N} = (*, *, \underline{\underline{0}})$ på sidorna, så

$x_3 (0,0,1) \cdot \vec{N} = 0$ och därmed

$$\iint_{\text{sidoyta}} (0,0,x_3) \cdot \vec{N} \, dS = 0.$$