STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2023-05-17

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Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

Consider a market with constant annual interest rate $R = 3\%$. The current time is $t = 0$.

(A) Consider a bond with face value $F = 100$, annual coupons $C = 20$ and maturity in 3 years. What is the current value of the bond? (5 p)

(B) Consider a person who deposits a certain amount C each year in a bank account (with the interest rate stated above). There are altogether 5 deposits.

How large does the deposit C have to be in order for the bank account to contain 400 000 SEK exactly 1 year after the last deposit is made? (5 p)

Problem 2

(A) Consider a market consisting of two stocks, with expected returns $\mu_1 = 0.2$ and $\mu_2 = 0.3$, and standard deviations of returns $\sigma_1 = 0.2$ and $\sigma_2 = 0.3$ respectively. The correlation between returns of the two stocks is $\rho_{12} = 0.3$.

Compute the minimum variance portfolio corresponding to these two stocks.

Hint:

$$w_{MVP} = \frac{uC^{-1}}{uC^{-1}u^T} \tag{1}$$

(5 p)

(B) State the optimization problem that corresponds to the minimum variance portfolio. Solve the problem by deriving an expression for w_{mvp} when considering only two stocks. Your derivation should not rely on using the formula in (1) explicitly.

(5 p)

Problem 3

Consider a European call option with strike price $X = 10$ and expiry at time 1, on an underlying with asset price process $S(t)$.

The financial model is a one-period binomial model $t = 0, 1$, with $S(0) = 10$, $U = 0.2$, $D = -0.1$, and $R = 0.1$ (risk-free interest rate with 1 year periodic compounding).

As usual we use the notation $S^u = S(0)(1 + U)$ and $S^d = S(0)(1 + D)$.

(A) Find the replicating portfolio $(x(1), y(1))$ for the call option. (5 p)

(B) Use the replicating portfolio to find the value of the call option, $C_E(0)$. (2 p)

(C) Find the value of the call option, $C_E(0)$, using the risk-neutral valuation formula. (3 p)

Problem 4

Suppose we have a one-period binomial model, with

$$R = 0.1, \;\; U = 0.2, \;\; D = -0.1, \;\; S(0) = 1.$$

Consider a put option with strike price $X = 1.1$ and expiry time $t = 1$.

(A) Find the value of the put $P_E(0)$ assuming it is European. (5 p)

(B) Find the value of the put $P_A(0)$ assuming it is American. (5 p)

Problem 5

Consider a European call option and a European put option. For both options hold that the exercise time is T and the strike price is X and the underlying share (which pays no dividends) is the same. The continuously compounded interest rate is r.

State and prove the corresponding put-call parity.

Hints:

(1) Recall that the put-call parity is that a certain relationship between $C_E(0)$, $P_E(0)$, X, $S(0)$, T and r holds.

(2) The proof may consist of identifying a suitable replicating portfolio (replicating a certain combination of the options).

(10 p)