MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Markus Hausmann

Tentamensskrivning i
Ordinary differential equations VT 2023
7.5 hp
24 May 2023

No calculators, books, or other resources allowed. The total score is 24 points. The subsequent oral exam has a maximum of 6 points. An overall total of 15 points plus a successful completion of the group project are required to pass.

Problem 1 (4 points)

Find all solutions to the differential equation $x'(t) + tx(t) = t^2 + 4t + 1$.

Problem 2 (4 points)

Use the power series method to find the solution to the initial value problem:

$$\begin{aligned} x''(t) - 2tx'(t) + x(t) &= 0 \\ x(0) &= 1 \\ x'(0) &= 0 \end{aligned}$$

Problem 3 (4 points)

Solve the following initial value problem via the Laplace transform:

$$\begin{aligned} x''(t) - x(t) &= t \\ x(0) &= 0 \\ x'(0) &= 0 \end{aligned}$$

(Hint: Recall that the Laplace transform of the function $f(t) = t$ equals $\frac{1}{s^2}$.)

Problem 4 (4 points)

Find a fundamental matrix for the homogeneous system $x'(t) = Ax(t)$ with

$$A = \begin{pmatrix} 7 & 0 & -1 \\ 0 & 2 & 0 \\ 4 & 0 & 3 \end{pmatrix}.$$

Problem 5 (4 points)

We consider the boundary conditions $y(0) = y(1), y'(0) = 0$ on the interval $[0, 1]$.

(1) Give an example (with proof) of a 2nd order linear ODE for which the associated boundary value problem has a unique solution.
(2) Give an example (with proof) of a 2nd order linear ODE for which the associated boundary value problem does not have a unique solution.

Problem 6 (4 points)

Consider the autonomous system

$$\begin{cases} x' = -3x + 5y \\ y' = (x - y)(\cos(x + y) - 2) \end{cases}$$

What are its equilibrium points? Determine for each of the equlibrium points wheter it is stable, asymptotically stable or unstable.