MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Jonathan Rohleder

Tentamensskrivning i
Ordinary differential equations VT 2023
7.5 hp
Aug 25, 2023

No calculators, books, or other resources allowed. The total score is 24 points. The subsequent oral exam has a maximum of 6 points. An overall total of 15 points plus a successful completion of the group project are required to pass.

Problem 1 (4 points)

Find the (unique) solution to

$$x''(t) + 2x'(t) - 15x(t) = 30t + 11$$

satisfying the initial values $x(0) = 1$ and $x'(0) = -4$.

Problem 2 (4 points)

Use the Laplace transform to find the solution to the following initial value problem:

$$\begin{aligned} x''(t) - 3x'(t) + 2x(t) &= e^{3t} \\ x(0) &= 1 \\ x'(0) &= 0 \end{aligned}$$

Problem 3 (4 points)

Find a fundamental matrix for the homogeneous system $x'(t) = Ax(t)$ with

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix}.$$

Problem 4 (4 points)

Consider the following boundary problem. Compute its associated Green's function and express the solution in terms of it.

$$\begin{aligned} y''(x) - y(x) &= x^4 + 1 \\ y(0) &= 0 \\ y(1) &= 0 \end{aligned}$$

Problem 5 (4 points)

For which $k \in \mathbb{R}$ and $L > 0$ does there exist a non-trivial solution on the interval $[0, L]$ to the equation $y''(t) + ky(t) = 0$ with $y(0) = y(L) = 0$? Prove your answer.

Please turn over!

Problem 6 (4 points)

(1) Show that the autonomous systems

$$\begin{cases} x' = y \\ y' = -x \end{cases}$$

and

$$\begin{cases} x' = y(x^2 + y^2) \\ y' = -x(x^2 + y^2) \end{cases}$$

have the same orbits. That is, show that their solution curves only differ by a reparametrization. (Hint: First describe the solution curves for the first autonomous system and then show that a reparametrization solves the second.)

(2) Compute the equilibrium points for both systems and determine whether they are stable, asymptotically stable or unstable.