STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2023-08-25

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Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

(A) Describe with words in a few sentences what a European call option is. (3 p)

(B) Describe with words in a few sentences what it means that a derivative is American. (3 p)

(C) Describe with words in a few sentences what a forward contract is. Describe shortly also futures contracts and how these differ from forward contracts. (4 p)

Problem 2

Consider a one-period model with two stocks.

The expected return of stock 1 is 0.05 and the standard deviation is 0.1. The expected return of stock 2 is 0.06 and the standard deviation is 0.12. The corresponding correlation is 0.2.

There is also a risk-free asset with return $R = 0.02$. Short selling is allowed.

(A) Write down the mean vector $\mathbf{m}$ and the covariance matrix $\mathbf{C}$ of the returns of the stocks. Calculate the weights of the market portfolio $\mathbf{w}_M$. (5 p)

(B) Calculate the weights of the portfolio of only risky assets with the smallest variance, $\mathbf{w}_{MVP}$. (5 p)

Problem 3

Consider the one-period binomial model for financial markets with $t = 0, 1$, $S(0) = 20$, $U = 0.1$, $D = -0.05$, and $R = 0$ (risk-free interest rate with 1 year periodic compounding).

For this market we consider also a European call option with strike price $X = 18$ and expiry at time 1.

(A) Find the replicating portfolio $(x(1), y(1))$ for the call option.

What is the cost of the replicating portfolio? (5 p)

(B) Now assume that a proportional transactions cost of 5% is incurred each time the stock is **sold** (there are however **no transaction costs when buying the stock**). Find the replicating portfolio $(x(1), y(1))$ for the call option.

What is the cost of the replicating portfolio? (5 p)

Problem 4

Consider a Black-Scholes financial market for a financial asset price $S(t), t \geq 0$. Let $r = 0.05$, $\mu = 0.1$, $\sigma = 0.2$ and $S(0) = 10$.

Find the value at risk VaR (with 95 % confidence level and a one year horizon) of a position consisting of 10 shares.

Hint: $N^{-1}(0.05) \approx -1.645$.

(10 p)

Problem 5

Consider the two-period binomial model for financial markets with $t = 0, 1, 2$, $S(0) = 20$, $U = 0.1$, $D = -0.05$, and $R = 0$. Consider also a European put option with strike price $X = 20$ and maturity at time 2.

(A) Find the current value $P_E(0)$ of the option using the method of risk-neutral valuation. (4 p)

(B) Find a replicating portfolio strategy for the option. (4 p)

(C) Does here exist an alternative strike price $X' > 0$ such that the corresponding value of the put option is given by $P_E(0) = 10$? If yes, does there exist more than one such strike price? (2 p)