STOCKHOLM UNIVERSITY
Department of Mathematics
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Re-sit exam
Mathematics for Economic and Statistical Analysis
MM1005, Fall term; 7,5 ECTS
9 November 2022

Instructions:
- During the exam you may not use any textbook, class notes, or any other supporting material.
- Non-graphical calculators will be provided for the exam by the department. Other calculators may not be used.
- In all solutions, justify your answers? communicate your reasoning. Use ordinary language where appropriate, not just mathematical symbols.
- Use natural language, not just mathematical symbols. Write clearly and legibly
- Mark your final answer to each question clearly by putting a a box around it.

Grades: Each solved problem is awarded by up to 10 points. At least 35 points would guarantee grade E, 42 for D, 49 for C, 56 for B and 63 for A. Note that the problems are not ordered according to the difficulty!

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1. Compute the Taylor polynomial of degree three for the function $f(x) = \ln(x^3+1)$ about the point $x_0 = 0$.

Solution Hence, the Taylor polynomial of degree three of the function f around the point $x_0 = 0$ is

$$P_3(x) = \frac{6}{3!}x^3 = x^3.$$

2. Calculate the integrals

$$\text{a)} \quad \int \frac{2x^3 - x^2}{x-1} \mathrm{d}x \qquad\qquad \text{b)} \quad \int_0^\infty x2^{-x^2+1} \mathrm{d}x$$

Solution a) Making the polynomial division one obtains that

$$\int \frac{2x^3 - x^2}{x-1} \mathrm{d}x = \frac{2}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x-1| + C \qquad \text{where } C \in \mathbb{R}.$$

b)

$$\int_0^\infty x2^{-x^2+1} \mathrm{d}x = \frac{1}{\ln 2}.$$

3. The expression

$$3x^2y + e^{x+y} + \ln(x+y) - e^y = 0$$

defines y as a function of x. What is the equation of the tangent line to $y(x)$ at the point $x = 0$?

Solution The tangent line to $y(x)$ at $x = 0$ is given by

$$P(x) = 1 - (1+e)x.$$

4. Find the value of the parameters a, b such that the function given by $f(x) = 2x^3 + ax^2 - 12x + b$ has a stationary point at $x = 1$ with $f(1) = 0$. Determine for which intervals is the function increasing/decreasing.

Solution The function is increasing on the interval $(-\infty, -2)$ and $(1, +\infty)$.

The function is decreasing on the interval $(-2, 1)$.

5. Consider the function

$$f(x,y) = 5 - 3x + 3y^2x + 2(x^3 + y^3).$$

(a) Find the stationary points of $f(x,y)$.

(b) Let D be the square with vertices $(0,0)$, $(0,1)$, $(-1,1)$ and $(-1,0)$. Determine the maximum and minimum value of f on D.

Solution The four stationary points are

$$\left(\pm\frac{1}{\sqrt{3}},\mp\frac{1}{\sqrt{3}}\right), \qquad \left(\pm\frac{1}{\sqrt{2}},0\right)$$

The maximum is 7 and it is attained at the point $(0,1)$, and the minimum is 5 and it is attained at $(-1,1)$ and $(0,0)$.

6. For which real numbers x is the series $S = \sum_{n\geq 1} \left(\ln\left(x+\sqrt{e}\right)\right)^n$ convergent? Find x such that $S = 1$.

Solution The series converges for $x \in (-\sqrt{e},,e-\sqrt{e})$

$S = 1$ only for $x = 0$.

(7) Consider the matrix $A = \begin{pmatrix} 1 & a & 6 \\ 2 & 4 & a \\ 1 & 2 & 0 \end{pmatrix}$ where a is a parameter.

(a) Calculate the determinant of A, $|A|$ as a function of the parameter a.

(b) Find all the values of a for which A is not invertible.

(c) Determine whether the system

$$\begin{cases} 2x+4y &= 2 \\ x+6z &= 1 \\ x+2y &= 1 \end{cases}$$

has one solution, no-solution or an infinite number o solutions. If it has any solution, find all possible solutions.

Solution $\det A = -2a + a^2 = a(a-2)$

So, for $a \in \{0,2\}$ te matrix A is no invertible.

The system has an infinite number of solutions.

GOOD LUCK!
