MATEMATISKA INSTITUTIONEN
STOCKHOLMS UNIVERSITET
Avd. Matematik
Examinator: Sofia Tirabassi

Tentamensskrivning i
Mathematics Methods for Economist MM3001
2023-08-15

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**Instructions:**

- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The use of graphic or programmable features is NOT permitted.
- The text is written in both English and Swedish, in case of discrepancies between the two the English version is the official one.
- You can use the formula sheet that come with the exam.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 3 Taylor polynomial of the function $f(x) = e^{x^2}$, around the point $x_0 = 0$, and use it to give an approximation of $f(0.1)$.

(2) Geometric Series: A construction firm has $\$K$ invested in a bank at a interest rate of $p\%$. They want to buy a new construction site. They need to pay $\$T$ at once and then $\$Y$ a year for $n$ years, with the first instance of payment after one year.
   (a) (1 pt) Give a formula that describes how much money is left on the account after 1 year and after 2 years.
   (b) (3pt) Let $K = 670000$, $T = 1000$, $p = 11.5$, $Y = 70000$ and $n = 12$ How much is the amount invested at the end of the payment period.
   (c) (1pt) Find a general formula that give you the amount left invested after $n$ years.

(3) Consider the function $f(x) = x^3 + 3x + 2$.
   (a) (2pt) Find where the function is increasing or decreasing, concave or convex.
   (b) (1pt) Find all the critical points and determine their type.
   (c) (1pt) Find the max and min value of the function on the interval $[0, 2]$.
   (d) (1pt) Compute $\lim_{x\to\pm\infty}$ and determine if the function has a max and/or a min in the interval $(-\infty, +\infty)$

(4) Compute the following integrals:
   (a) (2pt) $\int \frac{t}{t+5} + t^3 dt$,

(b) (3pt) $\int_0^1 (2x+1)^5 dy$.

(5) Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & 3 \\ 1 & k+2 & k+6 \\ -1 & 2 & k-3 \end{pmatrix}$$

(a) (2 pt) Compute the determinant of $A$, $|A|$ as a function of $k$.
(b) (1 pt) Find all the values of $c$ for which $A$ is not invertible.
(c) (2 pt) Set Now $k = 0$ and determine the number of solutions of the following linear system

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

(6) Consider the two variables function

$$f(x,y) = 2x^3 - 2xy + y^2 + 7$$

defined on the triangle

$$D = \{(x,y) \mid x \geq 0,\ -x+1 \geq y \geq 0, \}$$

(a) (2pt) Find all the critical points of $f(x,y)$ - even those lying outside $D$ and determine their type.
(b) (2pt) Determine the maximum and minimum points of $f$ on *boundary* of $D$. (In order to get credit you have to explain what you are doing, the correct answer without the right explanation will not be accepted)
(c) (1 pt) Determine the minimum and the maximum value of $f(x,y)$ on $D$. (In order to get credit you have to explain what you are doing, the correct answer without the right explanation will not be accepted)

GOOD LUCK!!!

**Senska texten, (formular finns ovanför)**

(1) (5pt) Beräkna grad 3 Taylor polinom till funktioner $f(x) = e^{x^2}$, omkring punkten $x_0 = 0$, och använda det för approximera $f(0.1)$.

(2) Geometriska Serier:
 (a) (3 pt) Bestämm för vilka $x$ den foljande konvergerar:
$$S(x) = 5 - 10e^{-x} + 20e^{-2x} - 40e^{-3x} + \cdots$$
 (b) (2pt) Bestäm om det finns $x$ sådan att $S(x) = \frac{1}{3}$.

(3) Betrakta funktionen $f(x) = (2x+1)e^{-x^2+1}$.
 (a) (2pt) Hitta alla de kritiska punkterna och bestäm dess typ.
 (b) (1pt) Bestämm var funktionen är vaxande och avtagende.
 (c) (1pt) Hitta den största och den minsta värden till funktionen i $[-1, 2]$.
 (d) (1pt) Räkna $\lim_{x\to\pm\infty} f(x)$ och skissa grafen till $f$.

(4) Räkna de följande integralerna:
 (a) (3pt) $\displaystyle\int \left(\sqrt{t}e^{\sqrt{t}} + \sqrt[5]{t^3}\right) dt,$
 (b) (2pt) $\displaystyle\int_0^1 \frac{3y}{y^2+1} dy.$

(5) Betrakta matrisen
$$A = \begin{pmatrix} 2 & 0 & 2+c \\ 3 & -1 & 0 \\ c & 0 & -2 \end{pmatrix}$$
 (a) (2 pt) Räkna determinanter till $A$, $|A|$ som en funktion av $c$.
 (b) (1 pt) Hitta alla värder $c$ sådan att $A$ inte är invertebär.
 (c) (2 pt) Räkna lösningen till system
$$\begin{cases} 2x & & -2z & = & 4 \\ 3x & -1y & +3z & = & 13 \\ -2x & & -2z & = & -8 \end{cases}$$

(6) Betrakta den foljande funktionen av två variabler
$$f(x,y) = e^{xy-x-y}$$
som defineras i trekanten
$$D = \{(x,y) \mid x \geq 0,\ y \geq 0,\ y \leq 4 - x\}$$
 (a) (2pt) Hitta alla kritiska punkter till $f(x,y)$ - punkter som ligger utanför $D$ också behövs att hitta.
 (b) (2pt) Hitta kandidater for storsta och den minsta punkter till $f$ på $D$ som ligger på *gränsen* av $D$.
 (c) (1 pt) Beräkna den storsta och den minst värden till $f$ på på $D$.

LYCKA TILL!!!