MATEMATISKA INSTITUTIONEN   Tentamensskrivning i
STOCKHOLMS UNIVERSITET   Mathematics Methods for Economist MM3001
Avd. Matematik   2024-03-13
Examinator: Sofia Tirabassi

Instructions:

- In case of ambiguity, one has to refer to the ENGLISH version of this exam.
- During the exam you MAY NOT use textbooks, class notes, or any other supporting material apart from the formula sheet given to you.
- Use of calculators is permitted for performing calculations. The only approved calculator are those that The use of graphic or programmable features is NOT permitted.
- Start every problem on a new page, and write at the top of the page which problem it belongs to. (But in multiple part problems it is not necessary to start every part on a new page)
- In all of your solutions, give explanations to clearly show your reasoning. Points may be deducted for unclear and wrong argument, even if the final answer is correct.
- Write clearly and legibly.
- Where applicable, indicate your final answer clearly by putting A BOX around it.

Note: There are six problems, some with multiple parts. The problems are not ordered according to difficulty

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(1) (5pt) Compute the degree 3 Taylor polynomial of the function $f(x) = x \ln(1 + (x-1)^2)$, around the point $x_0 = 1$, and use it to give an approximation of $f(1.1)$.

(2) Consider the curve in the plane given by the following equation

$$y^2 e^{2x} + C = 3y + x^2,$$

where C is an arbitrary constant.
   (a) (1 pts) Find the value of C such that the curves passes through the point $(0,3)$.
   (b) (2 pts) With $C = 0$ find the slope of the tangent to the curve in the point $(0,3)$.
   (c) (1 pt) Find the equation of the tangent to the curve in the point $(0,3)$.

(3) Consider the function $f(x) = e^{-x^3 + 30000x}$.
   (a) (2pts) Find where the function is increasing or decreasing. Find all the critical points and determine their type.
   (b) (1pt) Find the maximum and minimum values of the function on the interval $[-101, -99]$.
   (c) (2 pts) Compute $\lim_{x \to \pm\infty} f(x)$ and sketch the graph of $f(x)$.

(4) Compute the following integrals:
   (a) (2pt) $\int \left(\frac{\ln(t^5 + 1)}{t^5 + 1} t^4 + 4t^{\frac{7}{5}} \right) dt,$
   (b) (3pt) $\int_0^{+\infty} t e^{-t} dt.$

(5) Consider the matrix

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 8 & c & 2 \\ c & 6 & 1 \end{pmatrix}$$

    (a) (2 pt) Compute the determinant of A, $|A|$ as a function of c.
    (b) (1 pt) Find all the values of c for which A is not invertible.
    (c) (2 pt) Determine whether the following linear system has 1, 0, or infinitely many solutions. In case there is just one solution, find this.

$$\begin{cases} +2x & & +z & = & 1 \\ +8x & +y & +2z & = & 3 \\ x & +6y & +z & = & 3 \end{cases}$$

(6) Consider the two variables function

$$f(x, y) = 48xy - 32x^3 - 24y^2$$

    (a) (2pt) Find all the critical points of $f(x, y)$ and, if possible, determine their type.
    (b) (2pt) Consider now D, the rectangle $0 \le x \le 1$ and $0 \le y \le 1$. Determine the maximum and minimum values taken by $f(x, y)$ on the *boundary* of the D.
    (c) (1 pt) Determine the minimum and the maximum value of $f(x, y)$ on D.

<div align="center">GOOD LUCK!!!</div>

Svenska Texten

(1) (5pt) Beräkna det tredje graden Taylors polynom till funktionen $f(x) = x \ln(1+(x-1)^2)$, omkring punkten $x_0 = 1$. Använd det för att approximera $f(1.1)$.

(2) Antar kurven i planen med ekvation:
$$y^2 e^{2x} + C = 3y + x^2,$$
där C är en godtycklig konstant.
   (a) (1 pts) Hitta värdent till C sådan att kurver går genom punkten med koordinater $(0,3)$.
   (b) (2 pts) Med $C = 0$ beräkna lutningen till kurven i punkten $(0,3)$.
   (c) (1 pt) Ange ekvationer of den linjen som är tangenten til kurvan i punkten $(0,3)$.

(3) Betrakta funktionen $f(x) = e^{-x^3 + 30000x}$.
   (a) (2pts) Hitta var funktionen är växande eller avtagende. Hitta all kritiska punkter och bestem deras typ.
   (b) (1pt) Hitta den största och den minsta och minimum värden som funktionen tar i intervallen $[-101, -99]$.
   (c) (2 pts) Beräkna $\lim_{x \to \pm\infty} f(x)$ och rita grafen till $f(x)$.

(4) Beräkna de följade integralerna
   (a) (2pt) $\int \left(\dfrac{\ln(t^5 + 1)}{t^5 + 1} t^4 + 4t^{\frac{7}{5}} \right) dt$,
   (b) (3pt) $\int_0^{+\infty} te^{-t} dt$.

(5) Betrekta matrisen
$$A = \begin{pmatrix} 3 & 0 & 1 \\ 8 & c & 2 \\ c & 6 & 1 \end{pmatrix}$$
   (a) (2 pt) Beräkn determinanten till A, $|A|$ som en funktioner av c.
   (b) (1 pt) Hitta alla värden till c sådana att A int är invertibara.
   (c) (2 pt) Besätm hur många lösningen (1, 0 eller odliga många) har den följande system. Om det finns bara ett, räkna det.
$$\begin{cases} +2x & & +z & = & 1 \\ +8x & +y & +2z & = & 3 \\ x & +6y & +z & = & 3 \end{cases}$$

(6) Betrkata den följnande funktionen i tva variabler
$$f(x,y) = 48xy - 32x^3 - 24y^2$$
   (a) (2pt) Hittal alla den kritiska punkter och, om möjligt, bestäm deras typ.
   (b) (2pt) La D vara rectangeln $0 \le x \le 1$ and $0 \le y \le 1$. Beräkna den mstörsta och den minsta värden till $f(x,y)$ på *gränsen* till D .
   (c) (1 pt) Beräkna den största och minsta värden till $f(x,y)$ på D.

Lycka Till!!!