STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2024-05-23

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Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.
- Answers should be motivated (unless otherwise stated).
- Any assumptions should be clearly stated and motivated.
- Start every problem on a new sheet of paper.
- Clearly number each sheet with problem number and sheet order.
- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

Let the continuously compounded interest rate be $r = 0.1$. The present time is $t = 0$.

(A) Consider a bond that matures in 2 years with face value $F = 100$ and annual coupons $C = 10$ (the first coupon is paid exactly one year from now). Find the present value of the bond. (5 p)

(B) Consider a unit zero coupon bond (face value 1) that matures in 10 years. What is the bond worth 2 years from now? How many years after the present time ($t = 0$) will the bond be worth 0.9? (5 p)

Problem 2

(A) Explain in words what a replicating portfolio strategy is (in the context of derivative pricing in a multi-period financial market model). (4 p)

(B) Consider a market with two dates $t = 0, 1$. Let $V(0)$ denote the value of a portfolio. If the portfolio value $V(0)$ satisfies three conditions then it corresponds to an arbitrage opportunity. One of these conditions is $V(0) = 0$. State the other two conditions (in terms of mathematical expressions). (6 p)

Problem 3

Consider the two-period binomial model for financial markets with $t = 0, 1, 2$, $S(0) = 100$, $U = 0.1$, $D = -0.1$, and $R = 0.05$. Consider a European call option with strike price $X = 100$ and maturity at time 2.

(A) Find the current value $C_E(0)$ of the option using the method of risk-neutral valuation. (5 p)

(B) Find a replicating portfolio strategy for the option. (5 p)

Problem 4

We have two shares with return standard deviations given by σ_1 and σ_2. The covariance is denoted by c_{12}, and we assume that the corresponding correlation satisfies $\rho_{12} < 1$. Denote by s the weight in share 1 and by $1 - s$ the weight in share 2.

(A) Determine the minimum variance portfolio for

$$\sigma_1^2 = 0.06$$
$$\sigma_2^2 = 0.07,$$
$$c_{12} = 0.0021.$$

Hint: use the information provided in **(B)**. (5 p)

(B) Show that the minimum variance portfolio is attained when s is given by

$$s_0 = \frac{\sigma_2^2 - c_{12}}{\sigma_2^2 + \sigma_1^2 - 2c_{12}}.$$

(5 p)

Problem 5

Consider a Black-Scholes financial market. Consider a European derivative with two years left until maturity and the payoff function

$$10 I_{\{x<1\}}.$$

Describe/interpret the derivative in financial terms with one sentence.

Derive a pricing formula (as explicit as possible) for this derivative.

Hint: recall that the indicator function $I_{\{x<c\}}$ *takes the value* 1 *in case* $x < c$ *and the value* 0 *otherwise.*

(10 p)