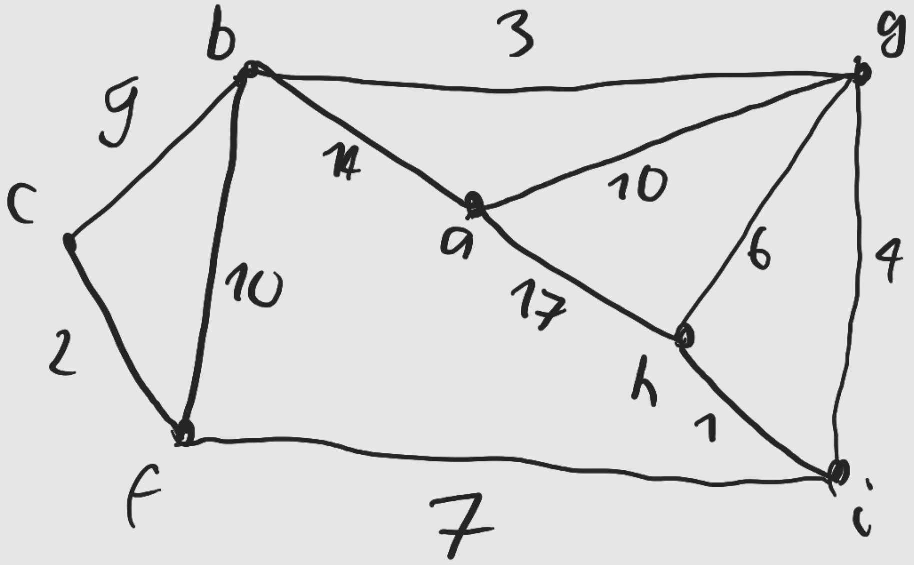
13.1 Exercise 2:

a) Apply Dijkstra's algorithm to the following graph and determine the shortest distance from vertex a to each of the other six vertices in G.

b) Determine a shortest path from vertex a to each of the vertices c, f and i.



Solution:

a) Step 1: $i = 0$ $\quad S_0 = \{a\}$

labels: $a: (0, -)$, $b: (\infty, -)$, $c: (\infty, -)$, $f: (\infty, -)$, $g: (\infty, -)$, $h: (\infty, -)$, $i: (\infty, -)$

Step 2: We look for each vertices in $\overline{S_0}$,
and update the labels if possible:

b: (14, a)   c: (∞, −)   f: (∞, −)

g: (10, a)   h: (17, a) , i: (∞, −)

Step 3: $b \in \overline{S_0}$ is not labeled (∞, −).

1) $L(g)$ is the minimum in $\overline{S_0}$

$S_1 = \{a, g\}$ $i = 1 < 6$ return to step 2.

Step 2: $a, g \in S_1$, and update the vertices in $\overline{S_1}$:

b: (13, g) , c: (∞, −) , f: (∞, −)

h: (16, g) , i: (19, g)

Step 3: The minimum is obtained at b

$S_2 = \{a, b, g\}$, $i = 2 < 6$

Step 2: $a, b; g \in S_2$, update the remaining vertices:

$c: (22, b)$, $F: (23, b)$, $h: (16, g)$

$i: (14, g)$

Step 3: $S_3 = \{a, b, g, i\}$, $i = 3 < 6$

Step 2: $c: (22, b)$, $f: (21, i)$, $h: (15, i)$

Step 3: $S_4 = \{a, b, g, h, i\}$, $i = 4$

Step 2: $c: (22, b)$, $f: (21, i)$

Step 3: $S_5 = \{a, b, f, g, h, i\}$, $i = 5$

Step 2: $c: (22, b)$

$S_6 = \{a, b, c, f, g, h, i\}$, $i = 6$.

The shortest distance are the following

b : 13

c : 22

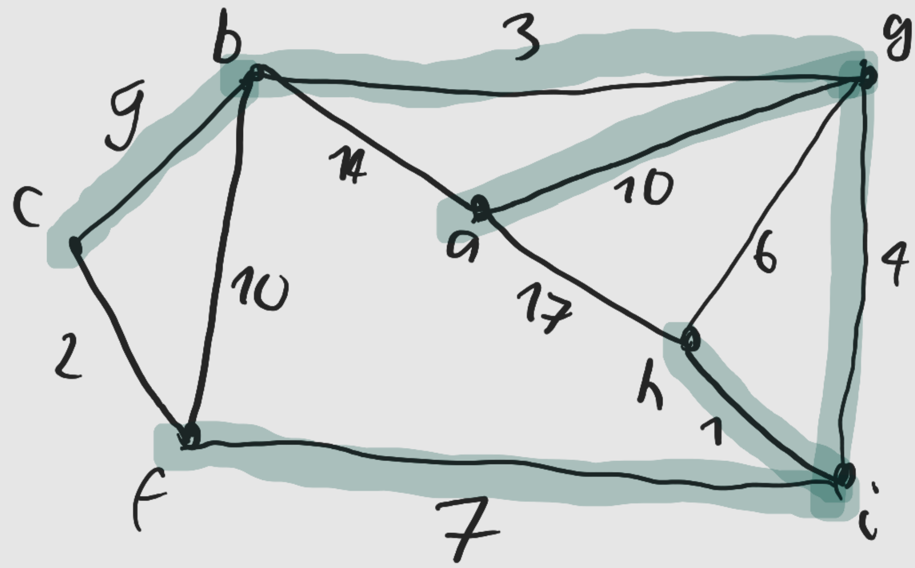
f : 21

g : 10

h : 15

i : 14

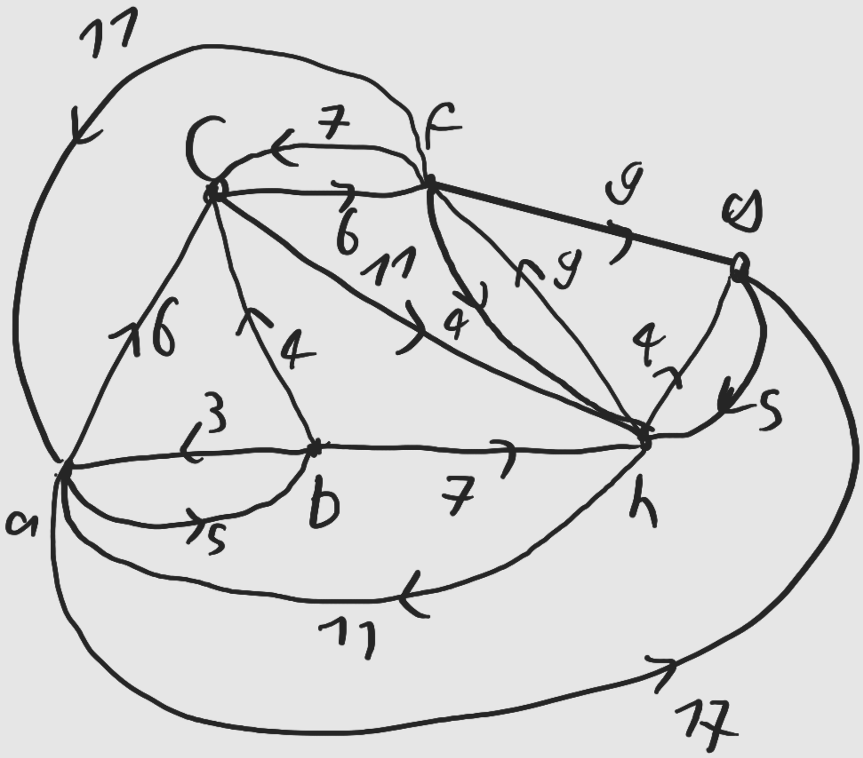
b)



are the shortest paths from a according to the algorithm.

Exercise 3:

Apply Dijkstra's algorithm to the following graph, starting from vertex a.



<u>Step 1</u>: $S_0 = \{a\}$ $i = 0$

labels: a: (0, –) , b: (∞, –) , c: (∞, –)

f: (∞, –) , g: (∞, –) , h: (∞, –)

<u>Step 2</u>: Update the labels

$\boxed{b: (5, a)}$ c: (6, a) , f: (∞, –)

g: (17, a) , h: (∞, –)

Step 3: $S_1 = \{a, b\}$ $i = 1$

Step 2: $\boxed{c : (6, a)}$, $f : (\infty, -)$

$g : (17, a)$, $h : (12, b)$

Step 3: $S_2 = \{a, b, c\}$, $i = 2$

Step 2: $\boxed{f : (12, c)}$, $g : (17, a)$,

$h : (12, b)$

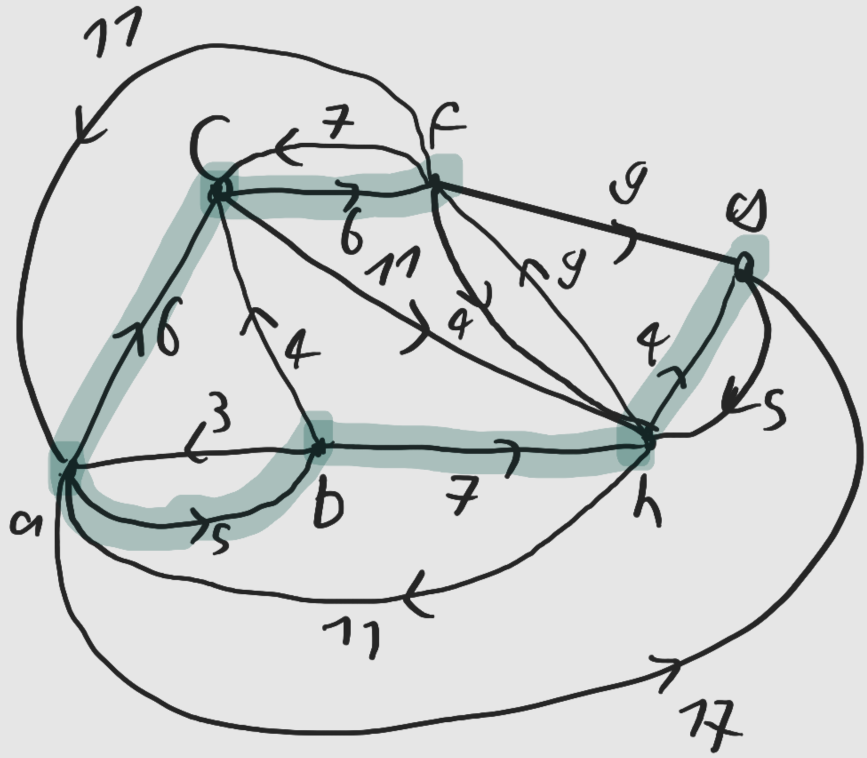
Step 3: $S_3 = \{a, b, c, f\}$ $i = 3$

Step 2: $g : (17, a)$, $\boxed{h : (12, b)}$

Step 3: $S_4 = \{a, b, c, f, h\}$, $i = 4$

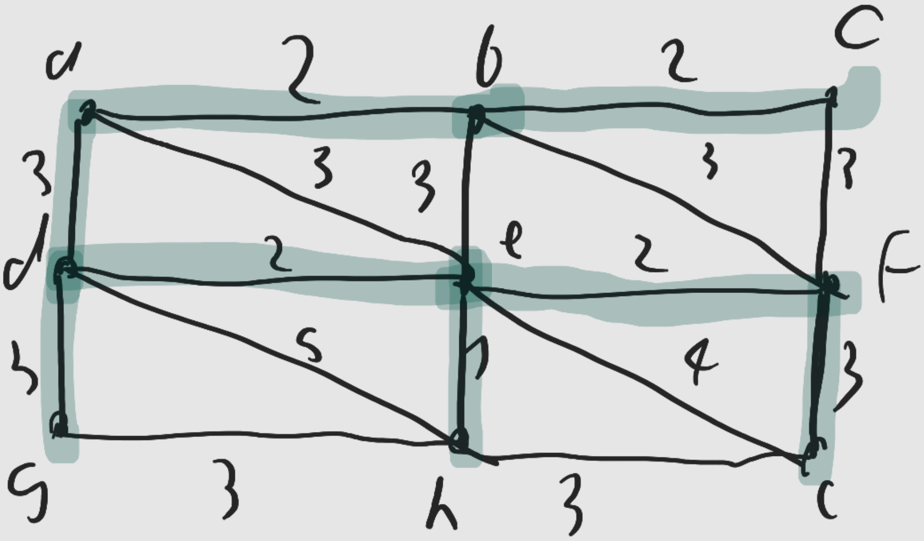
Step 2: $\boxed{g : (16, h)}$

b) Find a shortest path from vertex a to the vertices f, g, h :



13.2 Exercise 1:

Apply Kruskal's and Prim's algorithm to determine minimal spanning trees in the following graph:



Solution:

Kruskal:

Step 1: $i = 1$ $\quad e_1 = \{e, h\}$

Step 2: $\quad e_2 = \{a, b\}$

Step 2: $\quad e_3 = \{b, c\}$

Step 2: $\quad e_4 = \{d, e\}$

Step 2: $\quad e_5 = \{e, f\}$

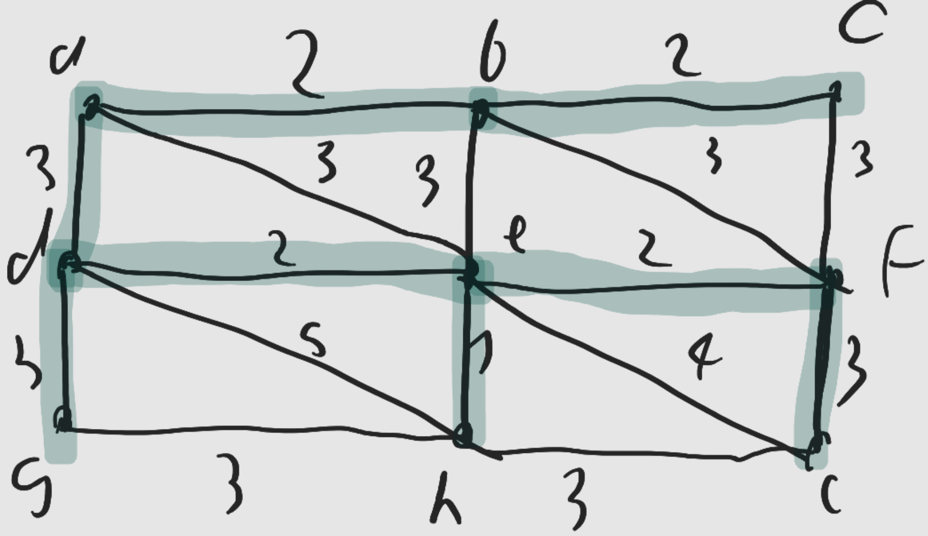
Step 2: $\quad e_6 = \{a, d\}$

Step 2: $\quad e_7 = \{d, g\}$

Step: $\quad e_8 = \{f, i\}$

Step 3: I now have an optimal spanning tree.

Prim:



Step 1: $i=1$

$P = \{a\}$ $\quad N = V - \{a\}$, $T = \emptyset$

Step 2: $\{a, b\}$ is an edge from P to N with smallest weight

$\rightsquigarrow P = \{a, b\}$, $N = V - P$

Step 2: $\{b, c\}$ has smallest weight

$\rightsquigarrow P = \{a, b, c\}$, $N = V - P$

Step 2: $\{a, d\}$ is an edge with smallest weight.

$P = \{a, b, c, d\}$, $N = V - P$

Step 2: $\{d,e\}$ has smallest weight

$P = \{a,b,c,d,e\}$, $N = V-P$

Step 3: $P = \{a,b,c,d,e,h\}$, $N = V-P$

⋮

we eventually get a spanning tree.

Exercise 4,

We have the following distances between cities, from table 1 and want to construct a minimal spanning tree.

We apply Kruskal's algorithm

$e_1 = \{B,I\} = 57$

$e_2 = \{B,T\} = 58$

$e_3 = \{G, S\} = 58$

$e_4 = \{F, S\} = 79$

$e_5 = \{E, T\} = 113$

$e_6 = \{F, I\} = 121$

Is the minimal spanning tree.

174 140 277

B, I, T, E connected

G, S, F connected

168 [illegible] $\boxed{121}$