STOCKHOLMS UNIVERSITET,
MATEMATISKA INSTITUTIONEN,
Avd. Matematisk statistik

Exam: Introduction to Finance Mathematics (MT5009), 2024-08-22

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Allowed aid: Calculator (provided by the department).

Return of exam: To be announced via the course webpage or the course forum.

The exam consists of five problems. Each problem gives a maximum of 10 points.

- The reasoning should be clear and concise.

- Answers should be motivated (unless otherwise stated).

- Any assumptions should be clearly stated and motivated.

- Start every problem on a new sheet of paper.

- Clearly number each sheet with problem number and sheet order.

- Write your code number (but no name) on each sheet.

Preliminary grading:

A	B	C	D	E
46	41	36	30	25

Good luck!

Problem 1

The current time is $t = 0$.

(A) Consider a bond maturing in 3 years. The face value is $F = 100$ and the annual coupons are $C = 10$. The continuously compounded interest rate is 2%. Calculate the present value of the bond. (5p)

(B) Consider a unit zero coupon bond (face value 1, no coupons) maturing at time $T = 2$. The current price is $P_0 = 0.9$. Find the continuously compounded interest rate r. (5p)

Problem 2

Consider the one-period binomial model. The current share price is $S(0)$. The share price at time 1 will be either $S(0)(1 + U)$ or $S(0)(1 + D)$. The risk-free interest rate (periodic compounding) is R. Suppose

$$R = 0, \ U = 0.1, \ D = -0.1, \ S(0) = 10.$$

Consider also a derivative with maturity at $t = 1$ and payoff function

$$f(s) = s^2.$$

(A) Determine the value of the derivative assuming it is European. (5 p)

(B) Determine the value of the derivative assuming it is American. (5 p)

Problem 3

Suppose a market consists of two shares whose returns are independent with expected values and standard deviations given by $\mu_1 = 0.2$, $\mu_2 = 0.1$, and $\sigma_1 = \sigma_2 = 1$. The risk-free rate is $R = 0$.

State the optimization problem that corresponds to the market portfolio $\mathbf{w}_M$ and solve this problem. State the solution $\mathbf{w}_M$ clearly. (10 p)

Problem 4

Consider the two-period binomial model with the following data

$$R = 0.05 \ U = 0.1, \ D = -0.1, \ S(0) = 100.$$

Find the value $P_E(0)$ of a European put option with maturity at $t = 2$ and strike $X = 90$. (10 p)

Problem 5

Consider an underlying asset that pays no dividends (and has no cost of carry) with the price process $S(t), t \geq 0$. The continuously compounded interest rate is a constant r.

Derive the forward price, for a forward contract initiated at $t = 0$, with delivery at time $T > 0$.

Give a short financial interpretation of the forward contract. (10 p)