Soft skills for Mathematicians

Tom Britton

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Preface

This book has been written when giving a PhD course at the Department of Mathematics and Stockholm University on 4 occassions between 2014 and 2024.

Thanks to ...

Chapter 1

Introduction

The aim of this text is to help young mathematicians to become successful academic scholars by improving their abilities in various aspects *beside* the most important: to master existing, and to develop new, mathematics. In particular I will describe my views on how to write a paper well (both a scientific paper and a more popular text), how to give an interesting oral presentation on a mathematical topic, how to write a (hopefully) successful application for a position, grant or similar, and how to act in the submission process of a manuscript submitted to a scientific journal in order to maximize the chance of acceptance.

I am of course not the first person to consider these aspects. There is quite a lot of literature, in particular on writing mathematics papers, and to some extent giving oral mathematics presentations. Further, many pieces of advice on these topics are generic for science, and there is of course extensive amounts written on these aspects for science in general and for other specific sub-discplines. But, even if many aspects of writing and presenting are general for all discplines I would claim that mathematics has some special charcteristics when it comes to writing and presenting (and applying for grants). The two main special features papers in mathematics are that the amount of text is typically much shorter, and secondly that it is much more specialized and hard to digest and understand. These two aspects have, according to me, consequences on how to write and present. My main philosophy in writing and presenting mathematics is an ambition to try to explain complicated matters such that more readers/listeners understand at least parts of the contents. Earlier, but much less now, there existed a snobbish culture in mathematics that wanted only the top experts to understand a

paper or presentation. There was no ambition to reach a bigger audience of even a small part of the contents. If you sympathize with this former culture the current text is probably not suited for you.

Who am I to write about this? I don't claim to be a trained specialist in technique for writing and presenting in the way that I am in mathematics. Still, having been in academia for over 30 years has given me some experience. I have written several textbooks, loads of scientific papers, given lote of scientific talks at conferences and in departments, but also in schools and other outreach activities, and I have applied for many grants and positions. I also have extensive experience from the "other side", i.e. evaluating papers, talks and applications: I am supervising Post Docs and PhD students, am associate editor for several journals, I have chaired a recruitment board at my University and have evaluated numerous applications for grants and positions.

Eventhough I hence have quite a lot (positive and negative) experience I still learn and there are several things I probably am not aware of or that could be described differently. As opposed to mathematics, writing and presenting has no one single correct "solution". There are many other texts dealing with writing and presenting mathematics, several which have inspired me and which are cited throughout the current text. My general advice is to look in several sources and to pick the suggestions that appeal to you.

My mathematical research area is towards the applied side (probability and statistical methodology with applications in medicine and biology). There are differences in writing styles between various subfields of mathematics. In several places in the text I discuss such differences. I have consulted with several "pure" mahematicians in order to avoid giving a biased point of view so I hope it is not too biased towards applied areas of mathematics.

A final important statement is that, just like mathematics, it is not enough to understand what is written in this or other texts. To become good you have to practice, so start writing papers, presenting talks and applying for grants and positions!

Chapter 2

Writing a scientific paper

If you are a mathematician in academia, then publishing papers in scientific journals is the most important output of your research achievements. Before you can write a paper, you need a mathematical problem and a (partial) solution to the problem, but this is not the focus of the current text, so we assume that you already have this and that you want to write a paper on the topic.

In order to write a good paper, the mathematical content is of course of utmost importance. Still I would say that this is not enough for a paper to become excellent or influential. It is also crucial that the paper is written in a clear pedagogical style, helping readers understand the contents at various levels of detail. How to do this is what the current chapter is about. As formulated by Halmos (1970, 2009): before starting to write you need to have something to say, an audience, and an outline. Halmos describes the writing of mathematical papers well: it is like writing in spirals.

There are many things to consider when writing a paper, some of more general character and some more about the details. We start with general features and after that move to details. Technical issues regarding which software to use and how to produce figures is not considered in this text.

2.1 Before starting to write

Suppose now that you are in the situation where you have some new mathematical result out of which you want to write a paper. You could start consider writing before all results have been obtained, but I suggest you

have most of them ready before starting to write. Otherwise it is hard to know the content of the paper. On the other hand, you may start writing before you have all minor things done, such as examples, numerical illustrations, production of figures and simulations. Beside the mathematical contents there are several other aspects to consider before you start writing.

2.1.1 Who should be authors and in what order?

One of the first things to decide upon is perhaps who should be author(s) of the paper. In general I would say that those who have contributed substantially to the work should be authors, but there is of course no formal definition of "substantial". In pure mathematics these questions are often more obvious in that there is a small group (1 to 3 people) who have done the work more or less jointly. In more applied areas of mathematics there may be scientists from different fields/expertise contributing in different ways (problem identification, modelling, model analysis, data collection, data analysis, ...). Another division between tasks is that of having more senior and more junior scientists on the project, supervisor and student being a typical example. In the latter case the supervisor may propose the problem and have some general idea on how to approach it, whereas the student may do the detailed analysis and most of the writing, which is later checked by the supervisor.

There is hence no obvious way to specify who should be author and who should not. Different organizations have slightly different stated criteria for being coauthor. The following ethical requirements for being coauthor of a mathematical paper are found on the website of the European Mathematical Society (shortened by me). They say the following regarding each coauthor:

- Significant contributions to the conception or design of the work; or the acquisition, analysis, or interpretation of data for the work; AND
- Accepts joint responsibility of the publication; AND
- Has approved to submit the paper.

They also state that individuals who have contributed significantly should be offered to be coauthor.

My advice is to discuss authorship in the beginning of a project (i.e. well before reaching the state of writing), having "significantly contributing to the work" as a guide to decide.

Individuals who have contributed to the work but not enough to be authors should definetely be mentioned in the acknowledgements – see Section 2.2.5.

Another related question is the order of authorship. Here there are different traditions in different subfields of science and even within Mathematical sciences. In (pure) Mathematics the tradition is to write authors in alphabetical order, with the interpretation that all authors have contributed jointly and it is hard to disentangle who has done the most "amount" or the most "essential" parts. This often happens when there are few authors, as is often the case in pure mathematics. An advantage with having alphabetical order is that it might prevent any disputes regarding who contributed most to the work. A disadvantage is that people with last name at the end of the alphabet nearly always belong to the "et al." group. In more applied parts of mathematics it is more common to list the authors differently. Then, typically, the first author(s) is the person who has done most of the work and writing, very often a PhD student or Post Doc. The last author(s) is often a more senior person, typically the supervisor, who might have suggested the problem and guided the progress of the project and given general advice on paper structure and similar. Then there might be other coauthors having done specialized parts such as collected data, programming, data analysis, and these would enter as middle authors.

Since there are different traditions in different subfields there will occur situations when it is not obvious which tradition to follow. One guide could then be to look at the journal where you aim to submit to, and to follow what seems to be the most common tradition of that particular journal. My recommendation is to have this discussion in the beginning of a project when discussing who should be authors of the paper. In particular in the non-alphabetical situation this will indicate who should be leading the project and the writing of the manuscript. I have some personal bad experiences where this discussion only came up towards the end, and where the authors had different opinions about whom should be the last author. I then gave up my point of view but it left me with a bad taste in my mouth ...

When submitting a paper to a journal there is another decision to be made: who should be the *corresponding author*. This implies being the one actually submitting the paper and having the correspondence with the journal in the revision process, so this involves some practical work. When having alphabetical order this is often done by a PhD student and in applied papers having different order of authorship it is often the first author (or

sometimes last author when submitting to prestigious journals). Choosing who should be corresponding author has never caused any conflict in my experience.

2.1.2 What are the main results of the paper?

Once you have obtained most results for the paper it is important to specify for yourself what the main results of the paper are. If you are several authors it is important to have this discussion among all authors. Specifying the main results is important since it is these things you want to convey to the reader. When there are several authors, in particular if you have different backgrounds, you might have different opinions regarding what are the main results, focusing either on proof techniques, connections to other theories or applications. Reaching an agreement at an early stage is fundamental since it will greatly influence how the manuscript should be written.

There should be between 1 to a maximum of about 5 main results in a paper. If you are not able to reduce it to this number you either need to think more about it, or to split up the contents into several papers.

I recommend to have the main messages written down early. They should be conveyed in several places in the paper: perhaps partly in title, definitely in the abstract, and in introduction/main result section and conclusion/discussion section. In other parts of the text, in particular in longer and more technical papers, it is wise to remind the reader of the purpose of the current part, often being a small brick towards one of the main results.

2.1.3 Which journal to submit to?

Once you have concluded what the main results are a second important question is what to do with the manuscript. The by far most common action is to submit to a scientific journal, but in certain situations you might aim for a monograph consisting of different contributions on some specified topic, and in areas like computer science conference proceedings of prestigeous conferences are most highly valued. In the latter case an additional important aspect could be that it should fit in, and possibly connect to, other papers of the monograph. As for journal you might at an early stage not want to decide exactly which journal, but it is important to at least decide what *type* of journal to submit to. Are you for instance aiming for a specialised journal covering a special area of mathematics, or a more general mathematics

journal, or a journal in which applications play a central role? This should affect how you write the paper so it has to be decided early on.

Later you will have to decide to which specific journal to submit to. My personal recommendation is to also do this quite early. For instance, many journals have a pre-specified structure for papers, specific LaTeX-templates and refereeing style, and so on. Another recommendation is to look in some papers of tentative journals to see the typical structure, writing style and contents of the particular journal.

How to decide which type of journal, and which particular journal to submit to is a delicate matter. If your work is on e.g. complex analysis the paper should of course not be submitted to algebra journals. Still, it remains to be decided if it should submitted to a journal on complex analysis, a more specialized journal within complex analysis, or a journal with more general scope, like mathematical analysis or even mathematics in general. Or if there is focus on some application suggesting that it should be submitted to e.g. a physics journal. A different categorization between journals is their quality. There is no unique way of defining quality, and "impact factor" is not a very useful concept in mathematics (as opposed to many other disciplines). Still I would say that there exist at least a partial quality ordering of many mathematics journals that nearly all mathematicians would agree upon. In my own area for instance, I don't know of anyone who would object to the statement that *Annals of Applied Probability* has higher quality than *Journal of Applied Probability* which in turn has higher quality than *Methodology and Computing in Applied Probability*. This does *not* mean that all papers in a high quality journal have a higher quality than those of a lower quality journal – it is a statement about the *average* paper quality.

My recommendation is to aim quite high when choosing journal, in particular when you think the result is of general interest. Upon deciding how high to aim for, you should discuss among coauthors and if you are a single author you might consult with a mentor, former supervisor, or other experienced colleague. It is silly to aim unrealistically high, like *Annals of Mathematics* for a typical PhD project, but on the other hand, to always publish in lower quality journals will make your research less influential, beside making your CV look less tractable. A disadvantage with aiming too high is the risk for disappointment if your manuscript is rejected by a journal, and also the delay to final publication. One aspect when deciding journal is therefore, at least when aiming high, possible information about average time to acceptance/rejection decision of different journals. I would say that

there is a tendency to publish in journals with wider scope the higher quality of a paper, but there are many exceptions to this and there are many specialized high quality mathematics journals. Personally I would however avoid submitting a manuscript to a low quality general mathematics journal (there may of course exist well-argued exceptions to this rule, but fewer). For example, I would not recommend submitting a manuscript to the (non-existing) journal *North-Western Swedish Journal of Mathematics*.

There are also an increasing number of "fake" scientific journals. By this I mean journals without any reviewing system where anyone who pays the publication cost gets to publish. Such journals, which often send out spam advertising e-mails, should of course be avoided.

Upon choosing which journal to submit to you might check the key references that your work is based on and related to. If many of them appear in a specific journal, or a couple of journals, this could be an indication where to submit. There is no rule that you should cite papers from the journal you submit to, but to me it looks reassuring when this happens.

Another aspect to consider is *open access*. If your paper has public access any potential reader can download it. If not then only those being connected to a university having subscription to the journal can do so (unless they pay). More and more universities and funding bodies require that scientists publish in open access journals. Quite often there is a journal fee for having the paper public access. Compared to the author salaries required to produce the results, the cost for public access is often negligible, but in some cases it might be hard to find this extra money. One advice is to check with your university library. Quite often the cost is waivered when the university subscribes to the journal, or it might be covered at a lower cost by the library. There is also a movement against publishers both having subscription cost *and* publication fee.

Having discussed various aspects to consider when selecting a journal to submit to, I would like to end by saying that the importance of journal choice has reduced somewhat the last 20-30 years or so. There are two main reasons to this: The first is that papers nowadays are nearly always found and read via search machines and university libraries typically have electronic mass-subscriptions to nearly all scientific journals. As a consequence, most papers are more or less equally easily found whether published in high or low quality journals. Secondly, most papers are put on ArXiv.org once or just before they have been submitted (see more about this and other submission issues in Chapter **??**). As a consequence, a somewhat longer time to publication

is less damaging nowadays, given that the result is available and dated once it appears on ArXiv. Having said this, the quality of the journal that you publish in is still of great importance. It is a quality check of your paper, and often publishing in high quality/profile journals indicates that you are an excellent scholar.

The choice of journal type will affect how you structure and write the paper. First, some journals may have specific style restrictions, but perhaps more important is the writing style. For instance, if a paper appears in a highly specialized journal there is less need to define terminology and notation as compared to when it appears in a more general journal. Further, a manuscript aimed for a general mathematics journal should preferably put the results into perspective also outside the special sub-discipline of the paper. Another aspect to consider is what to highlight: the results, the proof techniques or even the applicability? This will depend on the type of journal. My advice is to look in some (good) papers of the journal to see what things are emphasized. The most important aspects should typically appear prior to less central parts of the paper, the latter may appear just before the end of the paper, or even in an appendix (see more in Section 2.1.4).

2.1.4 The structure of the paper

One of the first things to decide upon when starting to write is how to structure the manuscript. If you are several authors, then everyone should be involved or at least have a saying on this. There is no single way to structure a paper, and how to structure it depends on the contents (technical vs applied, the length, the type of journal to submit to, and so on). Still, most mathematical papers have some things in common regarding structure. For example, all papers should have a title, abstract and reference list. Nearly always there is also an introduction as start of the main paper (after the abstract). In mathematics papers, an introduction may contain the most important results of the paper without giving all details. After the introduction there are often some more specific sections, the first possibly containing the main results. I recommend to end a paper with a short or long section called either Conclusions and/or Discussion. Here one can summarize the results again, put things in perspective, discuss possible alternative ways to approach the studied problems, and mention (new) open problems of interest for future study. Personally, I find it a bit odd if a paper ends without such a wrap-up, for example if a paper ends with the completion of a proof of some

lemma. A paper may also contain an Appendix (or in more applied journals denoted Supplementary material and being a separate file). This can contain technical proofs which are not central for the paper, various outputs/figures, code and tedious derivations. A good way to structure a paper is to write a skeleton of it and under each heading write a couple of sentences about what it should contain.

A theoretical paper and a more applied paper could hence be of the following forms:

Theoretical paper	**Applied paper**
Title	Title
Abstract	Abstract
1. Introduction and main results	1. Introduction
2 to e.g. 4: Specialized topics	2. Methods
5. Illustrations/Examples	3. Results
6. Discussion	4. Discussion
References	References

A special feature with papers in mathematics is that they are often quite complicated and time consuming to read. A very important aspect to consider when structuring the paper is that readers not having the time or capacity to digest the whole contents of the paper should still benefit from spending a few hours on it. Since most of us read a paper from the start, this means that a paper should not have an extensive literature overview, nor contain long proofs of preliminary results (e.g. lemmas), in the early parts of the paper. To me, a much more appealing structure is to, early on, motivate and define the problem under study, to briefly describe how you approached its solution and to state your main results of the analyses. Admittedly, this is not always easy. In applied journals such a paper structure is common and often supported by journal styles, but I encourage this philosophy also for theoretical papers. More specifically, this could mean that lemmas don't even appear in the beginning of a paper but instead the main results (typically theorems) enter quite early in the text. When proofs are an integral part of the paper the important proofs should also appear quite early in the text. But in such proofs early on in the text it is possible to refer forward in the text for some less central results/proofs, for example by writing: "... as Lemma 3.1 of the next section shows, the supremum ...".

Textbooks are often written in the traditional way of first stating and proving some preliminary results in lemmas, and only at the end of a section comes the main result in a theorem. This order has the advantage that it is probably the most logic order in that you hardly ever have to refer forward to currently unfamiliar (or at least un-proven) results. However, a big difference between textbooks and mathematics papers is that the former are mostly read by students who are forced to learning the material in order to pass a course. Very few people are forced to read mathematics papers. Of the ones who start, I believe that a bigger fraction will reach the main result and learn something from reading the paper, if it is structured the way I (and many others!) recommend, as opposed to using the textbook structure of presenting results. In the latter case more readers will stop reading before reaching the main parts of the paper (due to time restrictions, loss of interest, or other reasons).

When you structure the contents of a paper I think it is important to have in mind what I call the *Half-life rule*:

Half-life rule. Out of all people that read the title of a paper:

- 50% read the abstract
- 25% read the introduction
- Half of the readership is lost with each subsequent section (with the exception of the Discussion/Conclusion section, which might have slightly higher readership).
- $< 1\%$ read the appendices

If anything I think these numbers are *very optimistic*! Of course, these numbers are not fixed. One aim with putting efforts into the writing is to increase them ...

An important consequence of the half-life rule is that the title and abstract are very important: they should both aim at attracting the reader to continue reading, *and* they should convey the main results for those who do not continue. More about the title and abstract appear in Section 2.2.1.

The half-life rule is also a motivation to include the main results and methodologies in the introduction (succeeding a short background and statement of the problem). Have in mind that someone who only reads 1-2 pages of the paper should get something out of it.

To sum up, it is highly recommended to create a skeleton of the paper-to-be at an early stage. Such a skeleton could be a list of contents with section and subsection names containing very short descriptions where the different parts of the paper should appear and in what order. And have in mind that many readers will not reach the end of the paper before quitting to read, and this large group should also gain from reading your paper.

2.1.5 Notation

Another thing to decide upon at an early stage is notation. This is particularly crucial when several authors are involved, writing separate sections in parallel, but also otherwise it is recommended.

In many cases there are standards for notation. Follow such standards unless you have very good reasons not to – and mention these reasons in the text when you don't, since readers will be wondering. When introducing concepts without standard notation you should try to select notation that is easily remembered. For example, you can choose the latin or greek letter that corresponds to the first letter of the term (e.g. σ for some space and c for some constant).

Prefixes are useful but should be used in a consistent manner and not too much. If for example the closure of σ is denoted $\bar{\sigma}$ you should preferably not use the bar-notation of another symbol having a different meaning. Many papers have too much notation which prevents the reading to flow by frequently having to look back. Before introducing some notation you should always ask yourself: do I really need this notation or could I do without? If you use a lot of notation you might considering adding a table summarizing and explaining all notation.

2.2 Writing a first draft

Once you have considered the things of Section 2.1 (and of course have the mathematical contents) it is time to start writing. In the current section we present things to consider in the different parts of the article. I would wait before writing the abstract, and not think too much about the title (so only give a working title). Instead I recommend to start writing the introduction, or even better, one of the other main sections. You should have thought about the general structure within this section, perhaps with subsections

and listed things that should be contained in each subsection, and then write accordingly. I think it sometimes helps to skip some technical details when writing the first draft of a section in order to keep the logical flow. Later you can fill in these gaps.

In longer technical parts of the text it is important to occasionally remind the reader where you are aiming with the current part, so that the reader maintains awareness of how the current part connects to the main results. It is highly recommended to refer both backwards and forwards to help the reader understand how the various parts of the paper connect.

Another thing to have in mind when you write is that, probably, you know much more about the topic of the paper than nearly all readers! As a consequence, what is obvious to you need not be so for the typical reader, so be pedagogic and don't leave out logical steps in proofs or derivations. Sentences like "it easily follows that" or "an obvious consequence" should be prohibited unless of course they really are trivial to nearly all readers. It is much better to write "tedious but standard calculations reveal" or "after some further considerations being outside the scope of the current paper it can be shown that". Hopefully the reader believes you, and if not she may try to derive the results and feel a bit proud if she succeeds, and not ashamed if she doesn't. A reader who tries and fails to derive a result you claimed to be trivial will get angry and very likely stop reading your paper (and possibly other papers by you as well).

If you are writing a paper with other authors it is customary to have one person leading this process, and often also writing most of the material. Then some cloud system of sharing LaTex-files, such as Overleaf, is often recommendable. Even with such a tool I think it is best if different authors write sequentially rather than in parallell since it is important to know what is written earlier in a text when writing a new section. When writing with other authors notation should be discussed in advance and all authors need to adhere to the decided notation.

2.2.1 Title and abstract

Coming up with a good title is hard but important. There are several competing and contradictory features that should be fulfilled. First, the title should describe what the paper is about. Secondly it should convey the main result of the paper. Third it should attract potential readers to read the entire paper. Fourth and last, the title should be short and catchy. Sat-

isfying all these things is of course impossible, so these four features are more what to strive for.

Mathematicians are logical by nature which means they only want to state things in a precise way. This conflicts with writing a short, catchy and informative title. You will never be able to state all assumptions under which a result holds true in the title! Here you must be willing to skip many details. This is not the same as lying since you give the precise conditions in the paper.

While writing the paper, and in particular towards the end of the writing, you and your coauthors should think about different tentative titles. If you have decided upon a title early on in the writing process it is worthwhile returning to it at the end to see if it still is a good description of the paper contents.

To be specific, here are some examples of math-paper titles that I like: "Can one hear the shape of a drum?", "The Lorenz attractor exists", and "The critical probability of bond percolation on the square lattice equals 1/2", "The winner takes it all".

As for the abstract my recommendation is to write this only after the manuscript is written, at least the first draft of the manuscript should exist when you start writing an abstract. You should think of the abstract as a very condensed summary of the paper. Just like for the title, the purpose of the abstract is twofold: to motivate the problem and to convey the main results, as well as to attract readers to continue reading the manuscript.

Try to keep the abstract short — there are often word-count restrictions – and bring only up the very essential parts. A sentence or two describing the area and problem, followed by a few sentences on the results and what methodologies that were used. The abstract should hence also indicate what parts of mathematics are involved so that readers can understand if they belong to the suitable readership.

Once you have an abstract you should leave it aside, have others read it for comments, and also read it afresh again, and then revise it. An abstract should be polished several times given its importance.

A minor thing to have in mind is that an abstract and the rest of the paper should be able to read separately. In particular this holds for the abstract which appear on sites where the manuscript may not be accessible. A consequence is that references should be avoided in the abstract but if they are essential they need to be given in complete detail. Conversely, it is customary to have the main part of the text independent of the abstract.

You should hence not write "As described in the abstract ..." or similar in the main text.

2.2.2 Writing the Introduction

An introduction should start with a gentle presentation of the considered area and a motivation of the problem. My advice is to direct the introduction to a wider audience than the succeeding, possibly more specialized, sections. I don't mean that a layman should understand, but if you anticipate that only specialists in a very small sub-discipline of mathematics can digest the whole paper contents, you might aim at a bit wider group of mathematicians in the introduction.

It is also advisable to motivate *why* you study the problem at hand, for example by citing papers addressing related problems, describing consequences of solving the problem, giving an inspiring description of the problem, or similar. If you don't manage to interest the reader to the problem you have most likely lost their interest to continue reading.

In the early part of the introduction, where a presentation of the area is given, it is advisable to give some references. Perhaps one or two more general references for interested readers needing more background, but also some specific to where related problems and results are treated. I am however against having several pages of literature overview in the introduction. If such a literature overview is at all needed I recommend that it should appear in a separate section.

Key concepts should also be defined in the Introduction, whereas more detailed concepts and notation can wait to later sections.

As described earlier, I think that the main results of the paper should appear in the Introduction. These are hopefully interesting for the reader, and someone who stops reading after the introduction should have gained from reading it. It is also interesting for the reader to learn how the problems are analysed, so some words about the methodology in the paper is desirable in the Introduction.

An alternative to having a 2-3 page Introduction is to have a very short introduction followed by a section called Main results. With such a structure it might even be possible to give the essential parts of the proofs in such a section, at least at a heuristic level. Then people reading only these two sections would have learnt quite a lot.

It is customary to end the introduction by describing the structure of the rest of the paper in a short paragraph.

2.2.3 Writing other main sections and proofs

Before writing a section you should have made clear what it should contain and its logical structure and order.

The writing style of the paper will depend much on what the paper is about, to whom it is directed, and the purpose of each specific section. A general advice is that you should always have in mind that the purpose of writing is that you are helping the reader to understand some material, so be clear and pedagogical to the reader when you explain/prove things – otherwise you will loose them. When you are explaining complicated matters it might help to illustrate what you mean with a simple example. It is also important to, once in a while, remind the reader about the bigger picture so that they don't get lost in the details. You could for example write something like "The current lemma will be used when proving the convergence part of Theorem 3.1." or similar. You can also refer to the Introduction (or Main result) section explaining when a result has been mentioned earlier as a main result.

Most of the time some sections will contain proofs. Short proofs are usually quite easy to write and read. For longer proofs it is however important to help the reader. A good idea is to start such a proof with a description of the proof: "We start by proving x and y, and then use these results to prove z". Once x and y have been proven you should state this and say that you now prove z. Finally it is useful for the reader to summarize why and how you have proven the statement of the main result. Another way to describe the structure of a long and complicated proof is to present it in a graph (Halmos, 1971).

Some proofs contain a lot of equations. Remember that they are part of sentences by using connectives: therefore, thus, hence, we now see that, like ... so. If you make use of some definition or notation in a proof which was introduced much earlier in the paper it is nice to remind the reader about this, and to refer to where it was introduced.

In some situations it might not be clear to the reader why a result is of interest. Then you might have a highlighted and numbered "Remark" after the theorem/statement where you explain why it is important and how it connects to other results. Here you might also use more common words to

explain what the result really means and implies.

Once in a while you will realize the need for new notation. When this happens you should consider earlier notation and introduce new notation that fits, and only do this when necessary.

2.2.4 Writing Discussion/Conclusions

The final section is often called Discussion and/or Conclusions. This part typically starts by summarizing the main results once again. The main results will hence be mentioned, in different wordings, in four places of the manuscript: abstract, introduction, discussion, and of course where they are derived. There is nothing wrong with this – important messages should be repeated!

After summarizing the main results (in words) you might write a bit about alternative routes to approach the studied problem, and perhaps explaining why you chose your approach. You can also mention strengths and weaknesses with your work. A strength might be an elegant proof or a surprising result, and weaknesses could be that you were not able to derive some result or that lots of technicalities were needed. Admitting also weaknesses makes the reader feel confidence in you and your work.

Usually a paper does not solve all problems in an area. Having solved a problem also often give rise to new questions. It is hence important to include a discussion about new related unsolved problems. Hopefully this will trigger readers to continue your work which makes your efforts worthwhile, both for science in general, but also personally for you in that your work will get cited.

2.2.5 Appendices, Acknowledgements and References

An Appendix (or Supplementary Material) contains material that is not of main importance for the paper. In an applied paper this could be mathematical proofs of a result, where the result itself and its consequences for the application is the focus of the paper. In other more theoretical papers it might contain proofs of lemmas, or some examples/illustrations. If the paper also contains numerical illustration, then details on these parts could also appear in an appendix, as well as additional diagrams. The reason for putting material in an appendix, rather than having it in the main text is to keep the flow in the main text, and not to distract the reader from the main

material with less important details. The appendices usually appear just before or after the list of references (depending on the journal) and often have a different labelling system, such as A, B, ... In more applied papers appearing in applied journals, often have more susch material and for this reason they often appear as separate files and denoted Supplementary Material.

Each Appendix should be referred to in the main text. If an Appendix contains a mathematical derivation it should start exactly with the situation at hand where it is referred to, and end by the completion of its proof/derivation.

It is of course equally important that an Appendix is correct and clearly written. However, since much fewer will read it you might not spend as much time on polishing it to perfection as you should do with the main text.

Most papers have an "Acknowledgements". Here is the place to thank individuals that have contributed to the manuscript or that have been helpful in answering questions, giving advice, collecting data, or similar, but not to the extent that they should be coauthors of the paper. It is also customary to acknowledge financial support that enabled authors to work on the project (don't forget to give details such as the project number – some funding agencies will search for this when looking for outcome of funded projects). If, in the review process, you get suggestions from reviewers or editors which substantially improved the manuscript you should also thank them in the acknowledgements ("We thank two anonymous referees for pointing out many useful comments which greatly helped improve the paper"). This should not be done always, but only if their advice substantially improved the paper. The acknowledgement often appears just before the reference list, but this may vary between different journals.

When it comes to the reference list you should include publications that relate to your work, that address the problems treated in your work and publications that you make use of when obtaining your results. When you write a paper it is important that you make a literature study of the area of your work, and that you cite the relevant publications. This should preferably be done before you even start working on the project – it is important to know what others have already done. A paper leaving out important references in the area gives a very bad impression on referees and other readers familiar in the area. It happens that mathematicians at a late state find out about very related work. When such discovered work is too related or even overlapping with your work, this of course disappointing to say the least. If the other result in fact completely covers your result it threatens several months work

by you and coauthors. These sad events happen and there is not much to do except to try to find another direction on your work. Checking the literature before you start working on a problem of course reduces the risk of such a disaster. However, if the other work is related but not too much overlapping you can also take advantage of it. Such a publication implies that the problem area is of interest to others than you, so you should cite them. If you discovered the work at a late state you should claim that your's and the other's result are "independent" by writing something like: " ... see also recent independent work by Smith and Jones (2023) for a related result.".

Finally, all papers in the reference list must be referred to in the main text.

2.2.6 Citing within and to other papers

It is important to help the reader connect the different parts of your paper. One way to do this is by citing within your paper, both backwards and forward. You can write things like "That $f(x)$ is strictly increasing was shown in Theorem 2.2", and "... in Section 4.2 we will make use of this fact.".

There are different styles for numbering results like equations, theorems, definitions, and the like. One distinction is whether or not to label within each section (or even sub-section) like "Equation (3.2)", "Equation (3.1.2)" or "Equation (12)" and similar for theorems and such. The journal might have some guidelines here. If not, my general advice is to label within each section because it makes it easier to find it. Another question is if the various labels (equations, theorems, lemmas and similar) should be numbered separately or jointly. My personal opinion is that equations should always be labelled separately. When it comes to lemmas, theorems, corollaries and similar it is more a matter of taste and you should of course follow journal style in case there is one. An advantage with common labelling, so for example *Lemma 2.1* followed by *Theorem 2.2*, is that it is easier to find a specific result when referred to. On the down-side is that some readers may wonder where *Theorem 2.1* is to be found in the above example. When referring to equations, lemmas and similar, you should always use capital letter since these are names: "The main ingredient for proving Theorem 2.2 lies in Equation (2.10)". Personally, I prefer if the labelling is specific to the section of the paper, so Theorem 2.2 rather than Theorem 6, but this is a matter of

taste and also often prescribed by the journal.

A scientific paper should also refer to related work. If you write about mathematical results without citing important related references, then readers (and referees!) will get annoyed. For the same reason, it does not give a serious impression if your own references make up a major part of all references. You should avoid citing unpublished or hard-to-access references, and never cite unpublished work of your own. When you cite a paper you should always (!) check that the result you refer to really is stated in the paper – just because someone else cites it is not enough! When citing monographs or longer papers it is highly recommendable to cite a specific Chapter, Theorem or page number. This has two benefits: it helps the reader to find the cited result, but it also ensures that the result really appears in the reference! If several sentences or even paragraphs are taken from another source you must cite this source and explain that you are quoting it. If not you are committing an ethical misconduct! This also applies to the case when you are quoting your own publications.

There are different ways of citing other scientific papers or monographs. A common way is to write "... as shown in Kesten (1970)". If a result is important or original you may even write "... as shown elegantly by Kesten (1970)". If there are two authors you would write them both whereas if there are three or more you would write "... in Kesten et al. (1970)", where "et al." is a Latin abbreviation for et alia which means "and others". An alternative is to write " ... as shown in Kesten [7]" or simply "... has been shown in [7]", where the reference list is numbered and Kesten would be reference 7. Personally I prefer the author(s) followed by year, since it makes it easier to know the reference without looking in the list of references (from now on called the bibliography). Quite often the journal has a template for submissions which includes a specified reference style and specified bibliography style. As for the bibliography style the so-called Harvard referencing style is common in mathematical sciences. For papers appearing in journals, each reference starts with the last name of authors followed by there initial, then comes year within parenthesis, then article title followed by journal title in italics, and ending with journal volume and page number. Depending on the type of publication (journal paper, monograph, book chapter, ...) they are listed differently. There are also styles other than the Harvard referencing style, so check with the journal what style is to be used. In the present publication my intention is to use the Harvard referencing style, and to refer to a reference by last name followed by publication year, such as Pak (2018).

An important advice is to make use of the available facilities in LaTex. For instance, when you write the manuscript you should make use of the "label"-function for equations, sections, theorems and similar. If you are writing a longer paper it is wise to name the labels in a systematic way. You might give equations labels starting with "eq_..." and sections with "sec_...", where the dots should be replaced by some key word for the equations/section which you hopefully remember. When it comes to references it is common to keep all references (in the current paper but also other papers) that you use in a separate bib-file where you have also given labels to them. When writing the paper you can then simply refer to the labelled reference and include the whole file when type-setting, and only the references you cite will appear in the reference list. Exactly how this LaTex bibliography command works is best learnt by searching the web.

All theorems, lemmas and similar should be numbered whether or not you refer to them. Equations should however only be given a number if you refer to it somewhere.

2.2.7 AI tools and plagiarism

During the last few years a number of support tools for writing have appeared on the market, ChatGPT probably being the most well-known. It is not forbidden, and perhaps even recommendable, to make use such tools to improve the quality of a paper. It is however very important you are the author seeking help on specific matters from the support, rather than asking the support tool to write the paper for you. More and more scientific journals and organizations have ethical guidelines for how to make use of such tools, and what is not allowed. Clearly, you need to follow such guidelines when writing a paper.

Another related issue is to re-use figures. In general this is not allowed and falls under plagiarism which could be subject to misconduct of research. If you want to re-use a figure you first need to check if the publication where it appears has copyright of the figure. If there is no copyright problem you can in principle use a figure published elsewhere, but it is then very important that you clearly cite the original publiction where it appears: see Figure 3.1 originally from Diaconis et al. (2019). This is necessary also if you are the author and producer of the figure – if you don't site yourself it is self-plagiarism.

2.2.8 Some detailed advice

We now give some left-over detailed advice on writing mathematics. This is by no means a complete list of advice which is an impossible task.

In mathemathical papers results are often stated as lemmas, theorems, propositions and corollaries. The main results of a paper are denoted theorems. These are typically the results that the author(s) would like the reader to remember. Quite often some preliminary results are needed in order to build up and proving a theorem; these would be called lemmas. Results that are consequences of a theorem are denoted corollaries. The remaining category, Proposition, is slightly different. These are often some result which is not as important and might also be a bit less mathematically advanced. For the latter reason proofs are sometimes left out or only sketched, whereas proofs should always appear for the other three categories.

One thing to decide is whether to use "I" or "we" when writing the text. If there are several authors "I" is of course not an option, but I recommend to use "we" even when being a single author. I would for instance write "We now prove Theorem 4.1", where "we" would refer to me and the reader. I find it a bit boastful when writing things like "I now prove Theorem 4.1". There is no right or wrong here – that is why I use "I" in the current text which might seem to contradict what I just wrote. However, when it comes to personal opinions, as is the case in the current text, then using "I" comes naturally to me.

As in all writing, using active sentences is preferred compared to passive. Try to keep the language simple (but not boring) when you write a paper. Don't use too many abbreviations and don't use too much specialised terms that many readers are unfamiliar with. Remember that equations are part of sentences. You may use bold or italic words, but don't over-use them. A sentence should never start with a mathematical symbol, and don't write too many symbols or complicated expressions in running text. It is for example better to write "Pick an element x in X" than to write "Pick $x \in X$" (the reader will still translate the second sentence to the first in their mind when reading). Similarly, it is much easier for a reader if you write "S and T are the kernels of ϕ and ψ, respectively", rather than the more minimal-mathematical style "S (T) is the kernel of ϕ (ψ)".

Don't use phrases like: "it easily follows that ...". Better to write: "standard but tedious calculations reveal that ...". Admit, or give a reference, when logical steps are left out, otherwise readers might get stuck. Mathe-

matical equations are often best to display on separate lines unless they are very short and not referred to later.

Try to make the text "flow" and leave less relevant technicalities to appendices (or technical sections).

Another feature of a text is that it should be fairly easy to navigate in, and be pleasant to look at. The latter is another reason to why figures and tables are recommended. You should of course only include figures or tables when they have a message to deliver, but when they do they also make the paper more appealing to look at. All figures and tables must be referred to in the main text, typically expressing what they illustrate and the conclusions drawn from them. They should not be too complicated to understand, and the caption (below in figures and above in tables) should help the reader interpret the figure/table. Tables are often used for comparing different quantities. Make sure to have the quantities to compare as columns rather than rows – it is much easier for the eye to compare objects below each other rather than objects on the same row. Another tool for helping readers navigate in the paper is to use a sympol at the end of proofs (e.g. a solid black box). This shows where the proof ends and new text starts.

Perhaps it comes without saying, but you should make use of spell- and grammar-checks available in most softwares for writing LaTex, or large language models, such as ChatGPT, for suggestions of improvements in the writing (cf. Section 2.2.7).

There are of course many more detailed comments on writing mathematical texts and formulae than given above. Later in your career you build up your own experience but in the beginning you may search in other references, such as Knuth et al. (1989).

2.3 The "after-work"

We have now arrived at the third and final part of the writing stage: the after-work, the first two being preparation and writing. This is a time-consuming job and not always as stimulating as writing the first draft. However, no one writes a good paper in the first draft, so revising is essential!

After you and possible co-workers, have finished writing a first draft of the manuscript you should let the text rest for a few days. This will enable you to read the text more independently from the writing process, otherwise you might not react to strange formulations or even obvious typos. It is

probably a good idea to start editing in a copy of the first draft file so that you can return to the first draft if some parts are lost, or similar. A good idea in general is to, once in a while, date the current version and start editing a new copy of the LaTex file. Another advice is to make use of cloud services like Overleaf, in particular when writing jointly with several authors. These may also save earlier drafts of the manuscript enabling to return to older versions.

Once a few days have passed it is time to go through the manuscript thoroughly. When you return to the paper you should pretend for yourself that you are a typical reader of the paper. The first thing you should do when reading is to "zoom out" and look at the big picture. For example, you should look in the skeleton/paper structure which you produced before starting to write. Are all the main results there, are the derivations well structured and placed in a logical order, is the notation good,...? Lee (2010) proposes the use of a check-list when going through the first draft: are all assumptions stated, are the main results clear, ... Another thing to consider is the level of detail and complexity: is it suitable for the intended readership? I think it is more common that the manuscript is too compact and complex, but also the opposite makes reading time-consuming and unpleasant. Have a critical eye and ask yourself if any part can be removed: do all parts serve a purpose? If you in any of these situations think the text may be improved you should of course do this. If you for instance think a certain part needs additional explanation it is highly recommended to rewrite the whole explanation rather than just adding some parts in the middle of an existing explanation.

When you have looked at the bigger picture and made changes accordingly, it is time to read through the whole manuscript again, correcting any errors, inconsistencies, typos etcetera. Also here it is preferable to rewrite whole paragraphs rather than just a sentence, and don't always add material – removing some material might improve the paper even more!

If you are several authors you should also read the contributions of each other. Exactly how you revise should be discussed, but an advantage with revising each others' texts is that it might reduce the differences in writing style between authors and you may also correct bad writing habits of each other. My advice is that each author first revises their own section(s) once, thus removing obvious errors and mistakes. After that, you can then go through each others' sections and correct obvious typos and mistakes, and bring up bigger comments for discussion before revising.

After the whole manuscript has been scrutinised, my recommendation is

to leave it aside for another few days and repeat the procedure above one more time, but perhaps spending a bit less time on the different parts.

My strong recommendation, at least when writing papers in the beginning of your career, is to then let someone else read the manuscript and give comments. This could be a supervisor or mentor, or a colleague where you have might have agreed to read and comment each others' papers. Preferably the person reading should belong to the typical readership of your paper. It is important that you have revised it yourself first according to the instructions above before giving it to someone else to read: if I am asked to read and comment a manuscrit and find 5 typos on the first page I will get frustrated and not do the job well.

When you receive feedback you should modify the manuscript based on the comments you receive. If the reader finds some part complicated (or similar), then very likely other readers will as well.

Having gone through all these stages, the manuscript is now close to submission. Probably you (i.e. all authors) should read it through once again, but when this is done it is time to submit. Some students and young scientists try to improve a manuscript forever, often adding more and more material. Adding more material at a later stage is often a bad idea. Besides, you should not polish a manuscript forever. There will never be a 100% perfect version of your paper! Once the manuscript is good enough you should submit it. Most likely you will receive it back for some additional revision later anyway ... .

In the next chapter we discuss the actual submission process, including the process of revision.

Chapter 3

Literature

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