Galois Theory

SU — 2024 Spring Extra Practice 2

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§ Extra Practice §

Problem 1: Exercise that uses Sylow

Consider the polynomial $p(x) = x^7 - 20$ which is irreducible over $\mathbb{Q}$. Let L be the splitting field of $p(x)$ over $\mathbb{Q}$

(1) Show that $L/\mathbb{Q}$ is Galois and compute the degree of this extension.

(2) Show that there is a unique extension $\mathbb{Q} \subset K \subset L$ of degree 6.

(3) Show that there is a unique extension $\mathbb{Q} \subset K \subset L$ of degree 2.

Problem 2: Exercise that uses Sylow

Consider the polynomial $p(x) = x^{11} - 29$ which is irreducible over $\mathbb{Q}$. Let L be the splitting field of $p(x)$ over $\mathbb{Q}$

(1) Show that $L/\mathbb{Q}$ is Galois and compute the degree of this extension.

(2) Show that there is a unique extension $\mathbb{Q} \subset K \subset L$ of degree 11.

(3) Show that there is a unique extension $\mathbb{Q} \subset K \subset L$ of degree 2.

Problem 3: Exercise that uses Sylow

Consider the polynomial $p(x) = x^{13} - 15$ which is irreducible over $\mathbb{Q}$. Let L be the splitting field of $p(x)$ over $\mathbb{Q}$

(1) Show that $L/\mathbb{Q}$ is Galois and compute the degree of this extension.

(2) Show that there is a unique extension $\mathbb{Q} \subset K \subset L$ of degree 12.

(3) Show that there is a unique extension $\mathbb{Q} \subset K \subset L$ of degree 2.

Problem 4: Straight edge and compass

(1) Is the regular 24-gon contstructible with SE&C?

(2) **G** iven α a root of $x^7 - 2$. Is it constructible by straight edge and compass?