Ex 9.4.2 : a) $f(x) = 3e^{3x}$

$$= 3\sum_{k\geq 0} \frac{(3x)^k}{k!} = \sum_{k\geq 0} 3^{k+1}\frac{x^k}{k!}$$

So the sequence is $u_n = 3^{n+1}$.

b) $f(x) = 6e^{5x} - 3e^{2x}$

We get $u_n = 6\cdot 5^n - 3\, 2^n$.

c) $f(x) = e^x + x^2 \quad x^2 = 2!\cdot\frac{x^2}{2!}$

The sequence : $(1, 1, 1+2, 1, \dots, 1)$
$(1, 1, 3, 1, \dots, 1)$

d) $f(x) = e^{2x} - 3x^3 + 5x^2 + 7x$

The sequence is : $(1 = 2^0, 9, 14, -10, \dots, 2^n, \dots)$

e) $f(x) = \frac{1}{1-x}$

The sequence is : $u_n = n!$

f) $f(x) = \frac{3}{1-2x} + e^x$

The sequence is: $u_n = 3 \cdot 2^n \cdot n! + 1$

Ex 2.4.4: a) The exponential generating for one color: $e^x - 1$

For four colors: $(e^x-1)^4$

We look for the coefficient in $\frac{x^{12}}{12!}$.

b) We are looking for is the number of surjective maps $\{1, \dots, 12\} \to \{$white, red, blue, black$\}$

which is:

$$4^{12} - \binom{4}{1} 3^{12} + \binom{4}{2} 2^{12} - \binom{4}{3}.$$

c) The total number of blue and black flags is even if we are in one of two situations:

- the number of blue and black flags are even
- the number of blue and black flags is odd

The generating function in the first case is:

- $\left(\frac{e^{x}+e^{-x}}{2}\right)^{2} e^{2x} = \left(\frac{e^{2x}+2+e^{-2x}}{4}\right)e^{2x}$

$= \frac{1}{4}\left(e^{4x}+2e^{2x}+1\right)$

In the second case:

- $\left(\frac{e^{x}-e^{-x}}{2}\right)^{2} e^{2x}$

The coefficient in $\frac{x^{12}}{12!}$ for the first case:

$\frac{1}{4}\left(4^{12}+2^{13}\right)$

The coefficient in $\frac{x^{12}}{12!}$ for the second case is:

$\frac{1}{4}\left(4^{12}-2^{13}\right)$

The result is $\frac{4^{12}}{2}$.

Ex 3.4.6: a) The generating function for each case is:

i) $(1+x)^2\left(1+x+\frac{x^2}{2}\right)^2$

ii) $(1+x)\left(1+x+\frac{x^2}{2}\right)\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}\right)^2$

iii) $(1+x)^3\left(1+x+\frac{x^2}{2}\right)^4$

b) $(1+x)\left(1+x+\frac{x^2}{2}\right)\left(1+x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}\right)$

$\left(\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}\right)$

Ex 3.4.7: The generating function is:

$\left(\frac{x^3}{3!}+\frac{x^4}{4!}+\dots+\frac{x^{10}}{10!}\right)^4.$

Ex 9.5.2: a) (i) $f(x) = x$

(ii) $f(x) = \frac{x}{1-x}$

(iii) $f(x) = \frac{x}{(1-x)^2}$

(iv) $f(x) = \frac{x}{(1-x)^3}$

b) The sum $\sum_{k=1}^{n} k$ is given

by the coefficient in x^n in $\frac{x}{(1-x)^3}$

which is $\binom{n+1}{2} = \frac{n(n+1)}{2}$.

Ex 9.6.4: For the sequence:

$(a_0, a_0 + a_1, a_1 + a_2, \ldots a_k + a_{k-1})$

the generating function is

$x f(x) + f(x)$

For the second sequence we get

$$f(x) + x f(x) + x^2 f(x).$$

For the third sequence we get

$$\frac{f(x)}{4} + \frac{x f(x)}{2} + \frac{x^2 f(x)}{4}.$$