

MM5020 - Baseline theorems and topics for the oral exam

GROUPS

1. Definitions and basic property
2. Definition and order of D_{2n}
3. Inverse of an isomorphism is an isomorphism
4. Definition and properties of group homomorphism
5. Subgroups/subgroup criterion
6. Kernel, Images, Centralizers and Normalizers - what are there ?
7. Symmetric representation of a group action
8. Stabilizers and kernel of a group action
9. Normal subgroups - definition and equivalent characterizations
10. Construction of quotient (factor) group
11. H_1H_2 , definition and conditions for being a group.
12. First isomorphism theorem
13. Second isomorphism theorem
14. Third isomorphism theorem
15. Fourth isomorphism theorem
16. Orbit-Stabilizer theorem
17. Cayley's theorem
18. Class equations
19. Center of groups of order the power of a prime
20. Conjugacy classes in S_n
21. Automorphism, Inner automorphisms, characteristic groups
22. $\text{Aut}(\mathbb{Z}/n\mathbb{Z})$

23. Groups of order pq , with $p < q$ primes such that p does not divide $q - 1$
24. Sylow theorems
25. Application of Sylow theorems

RINGS

26. Rings, definitions and basic properties
27. $R[x]$, definition and special structure when R is an integral domain
28. Ideals and quotients
29. First Isomorphism theorem
30. Second isomorphism theorem
31. Prime and maximal ideals - definition and properties when quotienting by them
32. Various on ideals: intersection, products radical etc..
33. Localization in rings (construction of the field of fractions)
34. Euclidean domains, PID, UFD - definition and relations
35. Gauss Lemma

FIELD EXTENSIONS

36. Definition of field extension, structure as vector space, degree of an extension
37. Characteristic, size of finite fields
38. Algebraic, transcendental and finite extension, relation among them
39. Simple extension constructions and degree (definition of minimal polynomial).