Gauss' lemma:

let R be a UFD with field of fractions F and let $p(x) \in R[x]$. If $p(x)$ is reducible in $F[x]$ then $p(x)$ is reducible in $R[x]$.
(contrapositive: if $p(x)$ is irreducible in $R[x]$, then it is irreducible in $F[x]$).

More precisely, if $p(x) = A(x)B(x)$ for some non-constant polynomials $A(x)$ & $B(x) \in F[x]$, then there are non-zero elements r & $s \in F$ such that $rA(x) = a(x)$ and $sB(x) = b(x)$ both lie in $R[x]$ and $p(x) = a(x)b(x)$ is a factorisation in $R[x]$.

Example: The polynomial $x^2 \in \mathbb{Q}[x]$ can be decomposed as $x^2 = (2x)(\frac{1}{2}x)$, ~~where~~ so x^2 is not irreducible (i.e. it is reducible) in $\mathbb{Q}[x]$. Moreover, $x^2 = x \cdot x$ in $\mathbb{Z}[x]$, showing that it is also reducible in the smaller ring.

So if a polynomial in $\mathbb{Z}[x]$ is reducible, over $\mathbb{Q}$, then it is reducible over $\mathbb{Z}$.

Does the converse hold? (i.e. Suppose $p(x) \in \mathbb{Z}[x]$ is reducible in $\mathbb{Z}[x]$. Is it reducible in $\mathbb{Q}[x]$?)

No!, e.g. consider the polynomial $2x+2$.

We can write $2x+2 = 2(x+1)$, hence it is reducible in $\mathbb{Z}[x]$.

But it is irreducible over $\mathbb{Q}$, because 2 is unit in $\mathbb{Q}$. $(2 \cdot \frac{1}{2} = 1)$

Theorem: Let R be an I.D. Then R is a UFD iff $R[x]$ is a UFD.

One important consquence is about ring of polynomials in several variables.

Consider $\mathbb{Z}[x,y]$. We may view it as a ring ~~$\mathbb{Z}[x,y]$~~ of polynomials with coefficients in $\mathbb{Z}[x]$ over the variable y, i.e. we have

$$\mathbb{Z}[x,y] = \mathbb{Z}[x][y].$$

Now since $\mathbb{Z}[x]$ is UFD, $\mathbb{Z}[x,y]$ is so by the above theorem.

More generally, $R[x_1, \dots, x_n]$ is a UFD if R is a UFD.